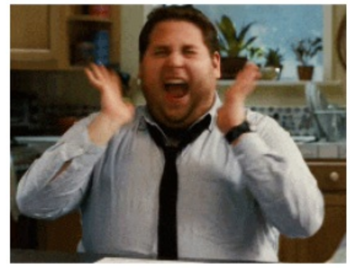
Math 3
Warm-up



State if the given functions are inverses.

1) $g(x) = 4 - \frac{3}{2}x$

$f(x) = \frac{1}{2}x + \frac{3}{2}$

not inverses

$y = 4 - \frac{3}{2}x$

$x = 4 - \frac{3}{2}y$

$-4 \quad -4$

$\left(-\frac{2}{3}\right)(x-4) = -\frac{3}{2}y\left(-\frac{2}{3}\right)$

$y = \frac{-2x+8}{3}$

2) $g(n) = \frac{-12 - 2n}{3}$

$f(n) = \frac{-5 + 6n}{5}$

not inverses

$y = \frac{-5+6n}{5}$

$5(n) = \left(\frac{-5+6y}{\cancel{5}}\right)\cancel{5}$

$5n = -5 + 6y$

$+5 \quad +5$

$\frac{5n+5}{6} = \frac{6y}{6}$

$y = \frac{5n+5}{6}$

3) $f(n) = \frac{-16 + n}{4}$

$g(n) = 4n + 16$

inverses

$y = 4n + 16$

$n = 4y + 16$

$-16 \quad -16$

$\frac{n-16}{4} = \frac{4y}{4}$

$y = \frac{n-16}{4}$ ✓

1.5 – Logarithmic Functions and Inverses

What is a logarithm?

A logarithm is the power to which a number must be raised in order to get some other number

power exponent power

$$\text{If } \underset{\text{power}}{y} = \underset{\text{base}}{b}^{\overset{\text{exponent}}{x}}, \text{ then } \log_{\underset{\text{base}}{b}} \overset{\text{power}}{y} = \overset{\text{exponent}}{x}$$

base base exponent

For example, the base ten logarithm of 100 is 2, because ten raised to the power of two is 100:

$100 = 10^2$ because **$\log_{10} 100 = 2$**

UP, DOWN, UP

If $y = b^x$, then

$\log_b y = x$

If $7 = 3^x$, then

$\log_3 7 = x$.

If $\log_b y = x$ then

$b^x = y$

If $\log_4 64 = 3$ then

$4^3 = 64$

Remember: If $y = b^x$ then $\log_b y = x$

If $25 = 5^2$ then

$\log_5 25 = 2$

If $729 = 3^6$ then

$\log_3 729 = 6$

If $1 = 10^0$ then

$\log_{10} 1 = 0$

If $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ then

$\log_{\frac{1}{2}} \frac{1}{8} = 3$

Let's pause for a second . . .

- If $y = b^x$ then $\log_b y = x$

- x in the exponential expression b^x is the logarithm in the equation $\log b_y = x$
-
- The base b in bx is the same as the base b in the logarithm

NOTE: b does not =1 and must be greater than 0
The logarithm of a negative number or zero is undefined.

Common Logs

- A common log is a logarithm that uses base 10. You can write the common logarithm $\log_{10}y$ as $\log y$

Evaluating Logarithms

Ex: Evaluate $\log_8 16$

$\mathrm{Log}_8 16 = x$	Write an equation in log form
$16 = 8^x$	Convert to exponential form
$2^4 = (2^3)^x$	Rewrite using the same base. In this case, base of 2
$\cancel{2}^4 = \cancel{2}^{3x}$	Power of exponents
$4 = 3x$	Set the exponents equal to each other
$x=4/3$	Solve for x

Therefore, $\mathrm{Log}_8 16=4/3$

Ex: Evaluate $\log_{64} \frac{1}{32}$

2^5 2^6

$$\log_{64} \frac{1}{32} = x$$

Write an equation in log form

$$\frac{1}{32} = 64^x$$

Convert to exponential form

$$\frac{1}{2^5} = 2^{6x} \rightarrow \cancel{2}^{-5} = \cancel{2}^{6x}$$

Rewrite using the same base. In this case, base of 2. Use negative expos!

-5 = 6x
x=-5/6

Set the exponents equal to each other
Solve for x

Therefore, $\log_{64} \frac{1}{32} = -\frac{5}{6}$

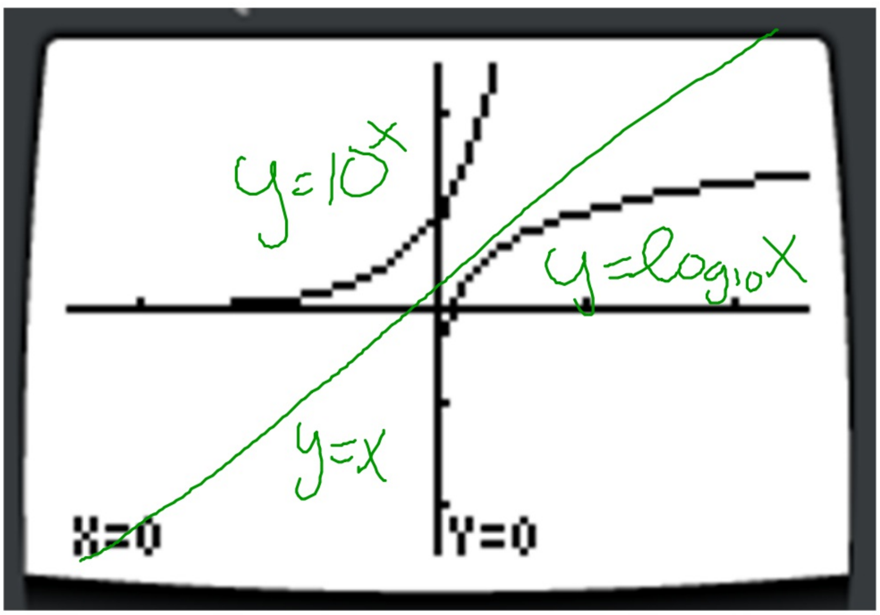
Let's try some

Evaluate the following:

$\log_9 27$	$\log_{10} 100$
$\log_9 27 = 1.5$	$\log_{10} 100 = 2$
$x = \frac{3}{2}$ $\quad 9^x = 27$	$10^x = 100$
$3^{2x} = 3^3$	
$\frac{2x}{2} = \frac{3}{2}$	

Graphs of Logarithmic Functions

- A logarithmic function is the inverse of an exponential function



In other words, $y= 10^x$ and $y=\log_{10}x$ are inverses of each other. Where is the line of reflection?

$y=x$

Let's try a more complicated one

Find the inverse of $y=\log_5(x-1)+2$

$y=\log_5(x-1)+2$	Start with the original function
$x=\log_5(y-1)+2$	Switch the x and y
$(x-2)=\log_5(y-1)$	Subtract 2 from both sides
$y-1=5^{(x-2)}$	Rewrite in $y=ab^x$ form
$y=5^{(x-2)}+1$	Add 1 to both sides

The inverse of $y=\log_5(x-1)+2$ is $y=5^{(x-2)}+1$

Let's try some

- Find the inverse of each function:

$Y=\log_{0.5}x$

$\cancel{x} = \log_{.5} y$

$y = .5^x$

$y=\log_5 x^2$

$x = \log_5 y^2$

$\sqrt{y^2} = \sqrt{5^x}$

$y = \sqrt{5^x}$

$= 5^{\frac{1}{2}x}$ or $5^{\frac{x}{2}}$

$y=\log(x-2)$

Hint: what is the base?

$x = \log_{10}(y-2)$

$y-2 = 10^x$

$+2 \quad +2$

$y = 10^x + 2$

Extra Practice:
-Find the Inverses

1.) $y = \log(-2x)$	2.) $y = \log_{\frac{1}{4}} x^5$
3.) $y = \log_{\frac{1}{5}}\left(x - 4\right)$	4.) $y = \log_3(4x - 4)$
5.) $y = \log_2\left(3x^3\right)$	6.) $y = -7\log_6(-3x)$