Math 3

-Warm-up

Find the inverse functions:

1.) $y = x^2 + 5$

$x = y^2 + 5$

$-5 \qquad -5$

$\sqrt{x-5} = \sqrt{y^2}$

$\boxed{y = \sqrt{x-5}}$

2.) $y = \frac{2}{3}x - 5$

$x = \frac{2}{3}y - 5$

$+5 \qquad +5$

$\frac{3}{2}(x+5) = \left(\frac{2}{3}y\right)\frac{3}{2}$

$\frac{3x+15}{2} = y$

$y = \frac{3}{2}x + \frac{15}{2}$

3.) $(x+3)^2$

$y = (x+3)^2$

$\sqrt{x} = \sqrt{(y+3)^2}$

$\sqrt{x} = y + 3$

$-3 \qquad -3$

$\boxed{y = (\sqrt{x}) - 3}$

4.) $y = \sqrt{x} + 8$

$x = (\sqrt{y}) + 8$

$-8 \qquad -8$

$(x-8)^2 = (\sqrt{y})^2$

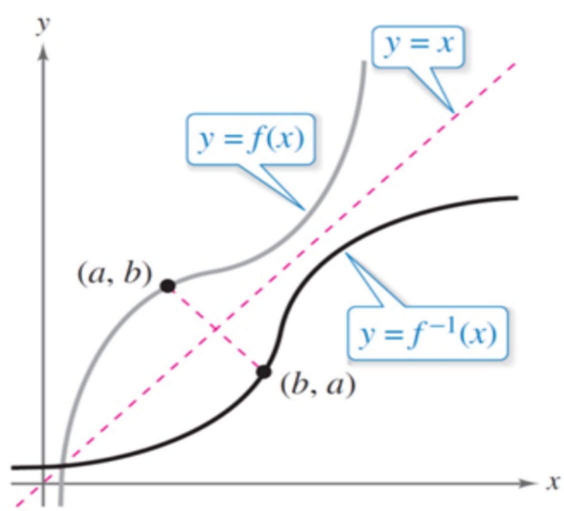
$\boxed{y = (x-8)^2}$

The Graph of an Inverse Function

The graphs of a function f and its inverse function f^{-1} *are related to each other in the following way.*

If the point (a, b) lies on the graph of f, then the point (b, a) must lie on the graph of f^{-1}*, and vice versa.*

This means that the graph of f^{-1} *is a reflection of the graph of f in the line y = x, as shown in Figure 2.37.*

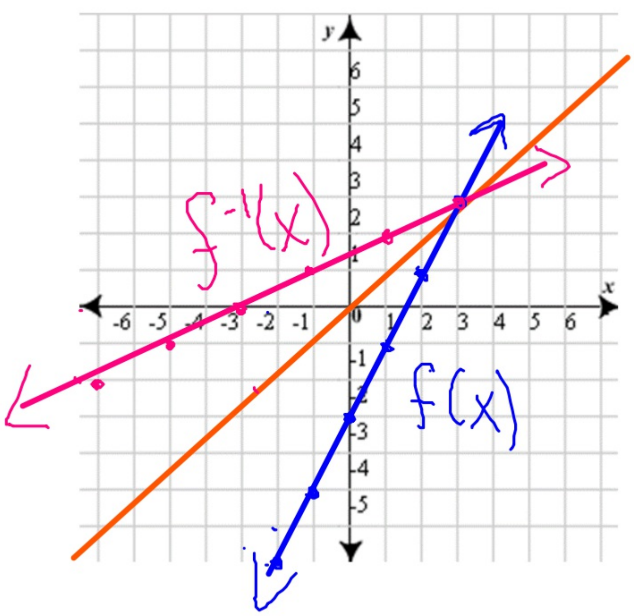


Sketch the graphs of

$f(x) = 2x - 3$ *AND* $f^{-1}(x) = \frac{1}{2}(x+3)$

show that the graphs are reflections of each other in the line y = x.

SKETCH THE INVERSE OF EACH FUNCTION



x	$f(x)$
-2	-7
-1	-5
0	-3
1	-1
2	1
3	3

x	$f^{-1}(x)$
-7	-2
-5	-1
-3	0
-1	1
1	2
3	3

PROVING INVERSES OF FUNCTIONS $f(3)=(3)4$
$=12$

Find the inverse of $f(x) = 4x$. Then verify that both $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are equal to the identity function.

$f(f^{-1}(x)) =$ **AND** $f^{-1}(f(x))$

$f(x)=4x$

$y=4x$

$\frac{x}{4}=\frac{\cancel{4}y}{\cancel{4}}$

$y=\frac{x}{4}$

$f\left(\frac{x}{4}\right)$

$=\cancel{4}\left(\frac{x}{\cancel{4}}\right)=x$ ✓

$f^{-1}(4x)$

$=\frac{\cancel{4}x}{\cancel{4}}=x$ ✓

EXAMPLES

Verify that f(x) and g(x) are inverse functions.

1) $f(x)=x^2$ $\quad g(x)=\sqrt{x}$ ✓

$y=x^2$

$\sqrt{x}=\sqrt{y^2}$

$y=\sqrt{x}$ ✓

3) $f(x)=\frac{x^3}{8}$ * $\quad g(x)=8\sqrt[3]{x}$

$y=\frac{x^3}{8}$

$8(x)=\left(\frac{y^3}{8}\right)8$

$\sqrt[3]{8x}=\sqrt[3]{y^3}$

$y=\sqrt[3]{8x}$

$y=2\sqrt[3]{x}$

not inverses

2) $f(x)=3-4x$ * $\quad g(x)=\frac{3-x}{4}$

$y=3-4x$

$x=3-4y$

$-3 \quad -3$

$\frac{x-3}{-4}=\frac{-4y}{-4}$

$y=\frac{x-3}{-4}$

$=\frac{-(x-3)}{4}$

$=\frac{-x+3}{4}$

$=\frac{3-x}{4}$

4) $f(x)=\frac{x+3}{x-2}$ $\quad g(x)=\boxed{\frac{2x+3}{x-1}}$

$f(g(x))$

$=\frac{\left(\frac{2x+3}{x-1}\right)+3}{\left(\frac{2x+3}{x-1}\right)-2}$

$\frac{\left(\frac{2x+3}{x-1}+\frac{3(x-1)}{x-1}\right)}{\left(\frac{2x+3}{x-1}-\frac{2(x-1)}{x-1}\right)}$

$\frac{2x+3}{x-1}+\frac{3x-3}{x-1}$

$\frac{\frac{5x}{x-1}}{\frac{5}{x-1}}$

$\frac{2x+3}{x-1}-\frac{2x-2}{x-1}$

$\frac{5}{x-1}$

$\frac{\cancel{5}x}{\cancel{x-1}}\cdot\frac{\cancel{x-1}}{\cancel{5}}=x$ ✓

$g(f(x))$

$\frac{2\left(\frac{x+3}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)-1}$

$\frac{\left(\frac{2x+6}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)-1}$

$\frac{\left[\frac{2x+6}{x-2}+\frac{3(x-2)}{x-2}\right]}{\left[\frac{x+3}{x-2}-\frac{(x-2)}{(x-2)}\right]}$

$\frac{2x+6}{x-2}+\frac{3x-6}{x-2}=\frac{5x}{x-2}$

$\frac{x+3}{x-2}-\frac{x-2}{x-2}=\frac{5}{x-2}$

$\frac{\cancel{5}x}{\cancel{x-2}}\cdot\frac{\cancel{x-2}}{\cancel{5}}=x$ ✓

Directions

1: Cut out all 24 of your puzzle pieces.

2: The 24 pieces will make 6 completed puzzles.

3: Show work in your notebook.

Each puzzle has:

- ∞1 piece showing a function
- ∞ 1 piece showing the function's inverse
- ∞ 1 piece showing the graphs of both functions
- ∞ 1 piece showing tables of both functions