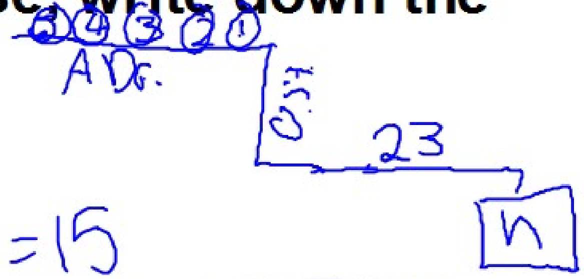
Math 3

Warm-up

Suppose you are given the following directions:

- From home go left on Route 23 for 5 miles
- Turn right onto Orchard St.
- Turn left onto Avon Dr.
- Tracy's house is the 5th house on the right.

If you start from Tracy's house, write down the directions to get home:



Solve:

2.) 8x-27-10-6x=15

$2x-37=15$
$+37 \quad +37$
$\frac{2x}{2}=\frac{52}{2}$
$x=26$

3.) -3(2x-3)=33

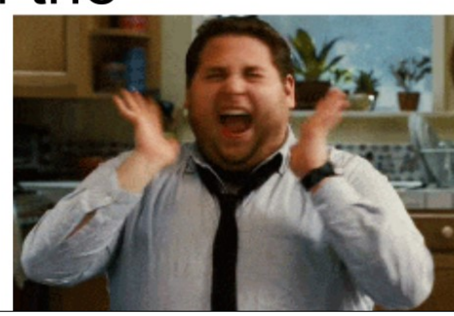
$-6x+9=33$
$-9 \quad -9$
$\frac{-6x}{-6}=\frac{24}{-6}$
$x=-4$

4.) -19+13x-11+2x=2

$15x-30=2$
$+30 \quad +30$
$\frac{15x}{15}=\frac{32}{15}$
$x=\frac{32}{15}$

Inverse Relations and Functions

An ___inverse___ function can be thought of as any function that undoes something that has already been done. In our everyday lives, examples of actions that are done and then undone could include turning off the faucet after it was turned on, taking off your jacket after putting it on, or even solving a puzzle and then taking it apart again. The inverse of an algebraic function can be used to solve problems for which the dependent quantity is known and the independent quantity is unknown.



Key Concepts

- Recall that a function is a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of *x*, there is exactly one value of *y*.
- function notation is the use of $f(x)$, which means "function of *x*," instead of *y* as the dependent variable in the equation of a function. For example, $f(x) = 2x + 1$ and $y = 2x + 1$ are equivalent functions.
- The inverse of a function, called an inverse function, is the function that results from switching the *x*- and *y*-variables in a given function.
- The inverse of $f(x)$ is written as $f^{-1}(x)$.

- The inverse function identifies all of the possible outputs of the function and the corresponding inputs. The outputs of a function are the independent values of the inverse of the function, and the inputs of the function are the dependent values of the inverse of the function.
- The domain of a function's inverse is the range of the original function.
- The range of a function's inverse is the domain of the original function.
- Consider a function with the following inputs and corresponding outputs. Notice that inputs and outputs of the function and inverse function are reversed.

Function	
Input	Output
–3	–17
–1	–11
1	–6
3	–3

Inverse	
Input	Output
–17	–3
–11	–1
–6	1
–3	3

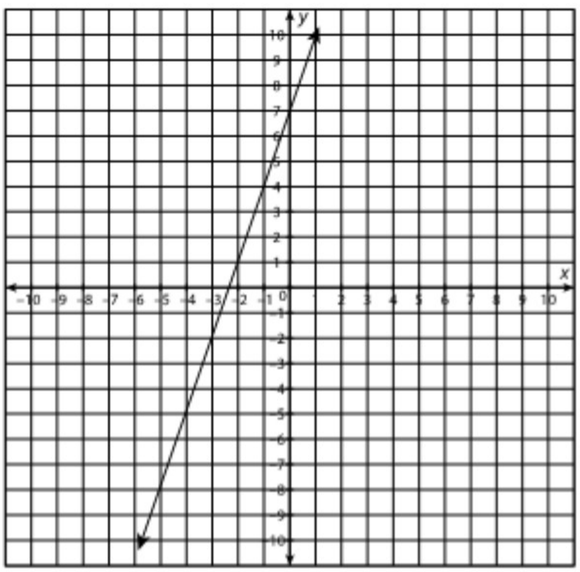
- To algebraically find the inverse of a function with *x* as the independent variable and *y* as the dependent variable, switch *x* and *y*, then solve for *y* in terms of *x* by using inverse operations, or operations that reverse the effect of other operations.

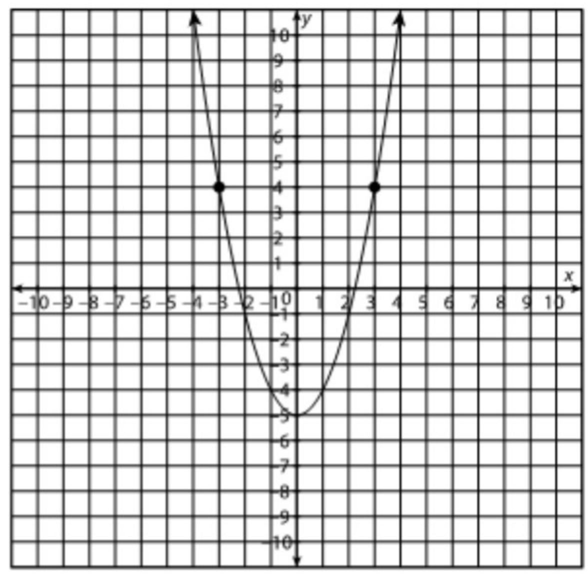
- The following table outlines the inverses of some common mathematical operations.

Original operation	Inverse operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication
Square	Taking the square root
Square root	Squaring the number

- In order to find the inverse of various functions, recall the following about linear and quadratic functions.
- A linear function is a first-degree equation that can be written in the form $f(x) = mx + b$, where m is the slope of the line and b is the y-intercept. The graph of a linear function is a straight line.
- The domain of a linear function is all real numbers.
- The range of a linear function is all real numbers.
- A quadratic function is in the form $y = a(x - h)^2 + k$, where a, h, and k are real numbers.
- The highest power of a quadratic function is 2.

- The domain of a quadratic function is all real numbers.
- The range of a quadratic function of the form $y = a(x - h)^2 + k$ has an extreme value at $y = k$. If $a > 0$, then the range is $y \geq k$. If $a < 0$, then the range is $y \leq k$.
- If a function is ____one-to-one____, meaning that each x-value is mapped onto exactly one y-value and each y-value is mapped onto exactly one x-value, then the inverse of the function is also a function.
- A linear function is one-to-one because each y-value is associated with a unique x-value.
- A quadratic function is a function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.
- A quadratic function is not one-to-one. Look at the diagrams on the next slide. Notice that in the parabola on the right, each y-value is not associated with a unique x-value, as shown by the two points.





- The domain of a function that isn't one-to-one can be restricted to find an inverse function.
- A quadratic function can be restricted to a domain of $x \geq h$ or $x \leq h$ in order for it to be one-to-one .
- If a quadratic function of the form $f(x) = a(x - h)^2 + k$ is restricted to a domain of $x \geq h$ or $x \leq h$, the range of the restricted function is equal to the range of the original function $f(x)$.
- The restricted domain of a function is the range of the function's inverse.

Guided Practice:

Example 1: Lana is driving home from her friend's house. She is driving at a steady speed, and her distance from her home, f(x), in miles, can be represented by the function

f(x) = –40x + 15, where x is her driving time in hours. Find the inverse function f –1(x) to show when, in hours, Lana will be x miles from home.

one-to-one ✓

$y = -40x + 15$

$x = -40y + 15$

$-15 \qquad -15$

$\frac{x-15}{-40} = \frac{-40y}{-40}$

$y = \frac{x-15}{-40}$ $y = \frac{-1}{40}x + \frac{3}{8}$

$f^{-1}(x) = \frac{-1}{40}x + \frac{3}{8}$

Example 2: Find the inverse function of $f(x) = 4x^2$. Use a restricted domain so the inverse is a function.

not one-to-one

restricted domain: $x \geq 0$

$y = 4x^2$ $\sqrt{y^2} = \sqrt{\frac{x}{4}}$

$\frac{x}{4} = \frac{4y^2}{4}$ $y = \pm \frac{\sqrt{x}}{\sqrt{4}} = \frac{\sqrt{x}}{2}$

$\boxed{f^{-1}(x) = \frac{\sqrt{x}}{2}}$

Find a table of values for each function and its inverse.

1. a. $f(x) = 3x + 1$ $3x+1$

Function	
x	f(x)
-1	-2
0	1
1	4
2	7
3	10
4	13
5	16

Inverse	
x	$f^{-1}(x)$
-2	-1
1	0
4	1
7	2
10	3
13	4
16	5

b. $f(x) = (2 - x)^2$ $(2-x)^2$

Function	
x	f(x)
-1	9
0	4
1	1
2	0
3	1
4	4
5	9

Inverse	
x	$f^{-1}(x)$
9	-1
4	0
1	1
0	2
1	3
4	4
9	5

Find the domain and range of the each function and the domain and range of its inverse in problems 2 (a-b) above.

a. $f(x) = \frac{1}{2}x + 1$ $\quad \frac{1}{2}x + 1$

$f(x)$ Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

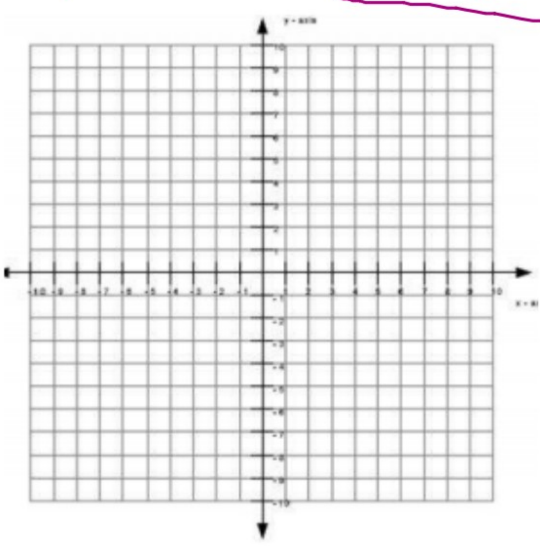
$f^{-1}(x)$ Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

$y = \frac{1}{2}x + 1$

$x = \frac{1}{2}y + 1$
$-1 \qquad -1$

$\frac{2}{1}(x-1) = \left(\frac{1}{2}y\right)\frac{2}{1}$

$\boxed{2x - 2 = y}$



b. $f(x) = (x-2)^2 + 3$ $\quad (x-2)^2 + 3$

$f(x)$ Domain: $x \le 2$ Range: $y \ge 3$

$f^{-1}(x)$ Domain: $x \ge 3$ Range: $y \le 2$

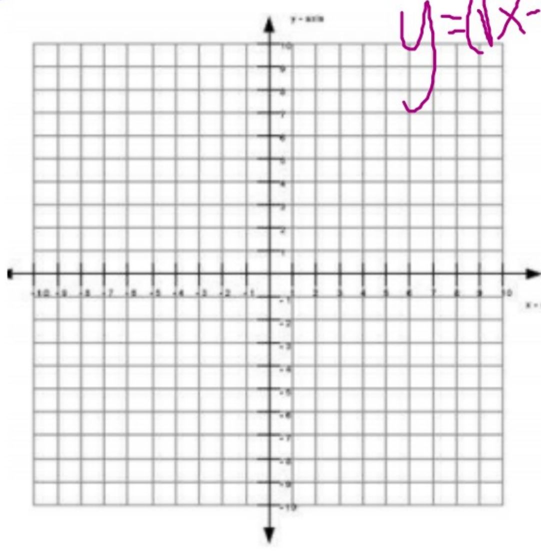
$y = (x-2)^2 + 3$

$x = (y-2)^2 + 3$
$-3 \qquad -3$

$\sqrt{x-3} = \sqrt{(y-2)^2}$

$\sqrt{x-3} = y - 2$
$+2 \qquad +2$

$y = \sqrt{x-3} + 2$



Find the inverse of each function below using the Flip and Find method.

a. $f(x) = 3x + 4$

b. $f(x) = (2x - 3)^2 - 1$

c. $f(x) = \frac{x+5}{-5}$

d. $f(x) = \sqrt{(x - 5)}$