

Warm-Up

Simplify the following expressions:



$$2\sqrt[3]{2xy}$$

$$10m^2\sqrt{2n}$$

$$-6\sqrt{3} + 3\sqrt{3} + 6\sqrt{5}$$
$$\boxed{-3\sqrt{3} + 6\sqrt{5}}$$

14) $\sqrt[3]{16xy}$

$$\sqrt[3]{8 \cdot 2 \cdot 2 \cdot x \cdot y}$$
$$\sqrt[3]{2^3 \cdot 2 \cdot x \cdot y}$$

10) $\sqrt[4]{200m^4n}$

$$\sqrt[4]{2 \cdot 100 \cdot m^4 \cdot n}$$
$$\sqrt[4]{2 \cdot 10 \cdot 10 \cdot m^4 \cdot n}$$

6) $-3\sqrt{12} + 3\sqrt{3} + 3\sqrt{20}$

$$-3\sqrt{4 \cdot 3} + 3\sqrt{3} + 3\sqrt{4 \cdot 5}$$
$$-3 \cdot 2\sqrt{3} + 3\sqrt{3} + 3 \cdot 2\sqrt{5}$$
$$-6\sqrt{3} + 3\sqrt{3} + 6\sqrt{5}$$



SIMPLIFYING

RATIONAL

EXPONENTS



SIMPLIFYING EXPRESSIONS WITH RATIONAL EXPONENTS

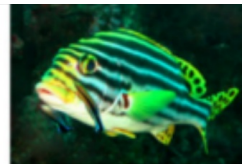
- Another way to write a radical expression is to use a rational exponent.
- Like the radical form, the exponent form always indicates the principal root.
- The word EXPONENT comes from the Latin word meaning "place outside."



Radical Form	=	Rational Form
$\sqrt{25}$	=	$25^{\frac{1}{2}}$
$\sqrt[3]{27}$	=	$27^{\frac{1}{3}}$
$\sqrt[4]{16}$	=	$16^{\frac{1}{4}}$



Simplify Each Expression:



a) $125^{\frac{1}{3}}$ = $\sqrt[3]{125}$ = 5

b) $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}$ = $5^{\frac{1}{2} + \frac{1}{2}}$ = 5^1 = 5

(Handwritten note: $5 \cdot 5$ with arrows pointing to the exponents)

c) $10^{\frac{1}{3}} \cdot 100^{\frac{1}{3}}$ = $\sqrt[3]{10} \cdot \sqrt[3]{100}$ = $\sqrt[3]{(10)(100)}$ = 10

(Handwritten note: 10 in a box)

Simplify each expression

$$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$$

x^1

$\sqrt{2}$

$$2^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$$

$$2 \cdot 2 = 4^{\frac{1}{2}} = \sqrt{4} = 2$$

$$x^2 \cdot x^3 = x^5$$

$$2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$$

$$\frac{1}{2} + \frac{1}{2} = 1 \quad 2$$



A rational exponent may have a numerator other than 1. The property $(a^m)^n = a^{mn}$ shows how to rewrite an expression with an exponent that is an improper fraction.



$$25^{\frac{3}{2}} = 25^{3 \cdot \frac{1}{2}} = (25^3)^{\frac{1}{2}} = \sqrt{25^3} = \sqrt{25 \cdot 25 \cdot 25}$$

or

$$25^{\frac{3}{2}} = 25^{(\frac{1}{2} \cdot 3)} = (25^{\frac{1}{2}})^3 = (\sqrt{25})^3 = \sqrt{25} \cdot \sqrt{25} \cdot \sqrt{25} = \sqrt{25 \cdot 25 \cdot 25}$$

Definition: Rational Exponents

If the n th root of a is a real number and m is an integer, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$: if m is negative then $a \neq 0$.

Convert to and from Radical Form

Rational to Radical:



$$6^{\frac{3}{4}} = \sqrt[4]{6^3} \text{ or } (\sqrt[4]{6})^3$$

index
radicand

The denominator is the radicand, the numerator is the exponent

Radical to Rational:

$$\text{a) } \sqrt[5]{7^4} = (\sqrt[5]{7})^4 = 7^{\frac{4}{5}}$$

$$\text{b) } 5^{-1.5} = 5^{-\frac{3}{2}} = \frac{1}{5^{\frac{3}{2}}} = \frac{1}{\sqrt{5^3}}$$

~~Be sure to follow the rules of exponents even with fractions!!!~~



Write the expressions $y^{-\frac{3}{8}}$ and $z^{0.4}$ in radical form.

$$y^{-\frac{3}{8}} = \frac{1}{y^{\frac{3}{8}}} = \frac{1}{\sqrt[8]{y^3}}$$

$$z^{0.4} = z^{\frac{2}{5}} = \sqrt[5]{z^2}$$

Write the expressions $\sqrt[7]{x^4}$ and $(\sqrt{y})^3$ in rational form.

$$x^{\frac{4}{7}}$$

$$y^{\frac{3}{2}}$$

Summary**Properties of Rational Exponents**

Let m and n represent rational numbers. Assume that no denominator equals 0.

Property

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$a^{-m} = \frac{1}{a^m}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example

$$8^{\frac{1}{3}} \cdot 8^{\frac{2}{3}} = 8^{\frac{1}{3} + \frac{2}{3}} = 8^1 = 8$$

$$\left(5^{\frac{1}{2}}\right)^4 = 5^{\frac{1}{2} \cdot 4} = 5^2 = 25$$

$$(4 \cdot 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 2 \cdot 5^{\frac{1}{2}}$$

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$$

$$\frac{\pi^{\frac{3}{2}}}{\pi^{\frac{1}{2}}} = \pi^{\frac{3}{2} - \frac{1}{2}} = \pi^1 = \pi$$

$$\left(\frac{5}{27}\right)^{\frac{1}{3}} = \frac{5^{\frac{1}{3}}}{27^{\frac{1}{3}}} = \frac{5^{\frac{1}{3}}}{3}$$



You can simplify a number with a rational exponent by using the properties of exponents or by converting the expression to a radical expression.

4**EXAMPLE****Simplifying Numbers With Rational Exponents**

Simplify each number.

a. $(-32)^{\frac{3}{5}}$

Method 1

$$\begin{aligned} (-32)^{\frac{3}{5}} &= ((-2)^5)^{\frac{3}{5}} \\ &= (-2)^{5 \cdot \frac{3}{5}} \\ &= (-2)^3 \\ &= -8 \end{aligned}$$

Method 2

$$\begin{aligned} (-32)^{\frac{3}{5}} &= (\sqrt[5]{-32})^3 \\ &= (\sqrt[5]{(-2)^5})^3 \\ &= (-2)^3 \\ &= -8 \end{aligned}$$

b. $4^{-3.5}$

Method 1

$$\begin{aligned} 4^{-3.5} &= 4^{-\frac{7}{2}} \\ &= (2^2)^{-\frac{7}{2}} \\ &= 2^{2 \cdot -\frac{7}{2}} \\ &= 2^{-7} \\ &= \frac{1}{2^7} \\ &= \frac{1}{128} \end{aligned}$$

Method 2

$$\begin{aligned} 4^{-3.5} &= 4^{-\frac{7}{2}} \\ &= \frac{1}{4^{\frac{7}{2}}} \\ &= \frac{1}{(\sqrt{4})^7} \\ &= \frac{1}{2^7} \\ &= \frac{1}{128} \end{aligned}$$



Simplify Each Expression:



a) $25^{-\frac{3}{2}}$ $\frac{1}{25^{\frac{3}{2}}} = \frac{1}{\sqrt{25^3}} =$

$\frac{1}{(25 \cdot 25) \cdot 25} = \frac{1}{25 \cdot 25} = \frac{1}{125}$

(Handwritten notes: 5·5, 5·5, 1/125 circled)

b) $32^{\frac{3}{5}}$

$\sqrt[5]{32^3} = \sqrt[5]{32 \cdot 32 \cdot 32} = 2 \cdot 2 \cdot 2 = 8$

√T = L

c) $(-32)^{\frac{4}{5}}$

$\sqrt[5]{-32^4} = \sqrt[5]{(-32)(-32)(-32)(-32)}$

$2 \cdot 2 \cdot 2 \cdot 2 = 16$

To write an expression with rational exponents in simplest form, write every exponent as a positive number.

$(8x^{15})^{-\frac{1}{3}}$

$[(16y^{-8})^{-\frac{3}{4}}]$

$8 \left(\frac{3}{4}\right)^{\frac{24}{4}} = 6$

$\left(\frac{16}{y^8}\right)^{-\frac{3}{4}}$

$\frac{16^{-\frac{3}{4}}}{(y^8)^{-\frac{3}{4}}} = \frac{(y^8)^{\frac{3}{4}}}{16^{\frac{3}{4}}}$



$\sqrt[4]{16 \cdot 16 \cdot 16}$

$4 \cdot 4 \cdot 4 \cdot 4$

$2 \cdot 2 \cdot 2 \cdot 2$

$\frac{y^6}{\sqrt[4]{16^3}}$

$\frac{y^6}{\sqrt[4]{8}}$

On your white board, work each problem.

a) $27^{\frac{1}{3}}$

b) $36^{\frac{1}{2}}$

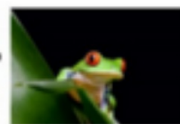
c) $2^{\frac{1}{2}} \cdot 32^{\frac{1}{2}}$

d) $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}}$

e) $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}}$



Write each expression in simplest form.
Assume that all variables are positive.



$$\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)^{-6}$$

$$\left(-27x^{-9}\right)^{\frac{1}{3}}$$

$$\left(x^{\frac{2}{3}}y^{-\frac{1}{6}}\right)^{-12}$$

$$\left(\frac{x^3}{x^{-1}}\right)^{-\frac{1}{4}}$$

$$\left(\frac{x^2}{x^{-11}}\right)^{\frac{1}{3}}$$

Activity Time



Partner Practice

1. Rational Exponents worksheet

- Work in pairs.
- All the odd numbers are due at the end of the class.
- What you don't finish is your homework.
- Talking volume < Music volume.

