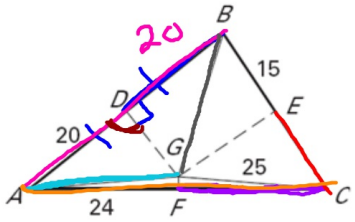
Math 3
Warm-Up

In the diagram, the perpendicular bisectors (shown with dashed segments) of ΔABC meet at point G--the *circumcenter*. and are shown dashed. Find the indicated measure.

11. AG = 25
12. BD = 20
13. CF = 24
14. AB = 40
15. CE = 15
16. AC = 48
17. m∠ADG = 90°

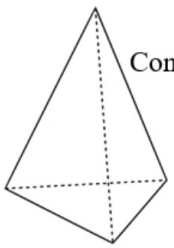


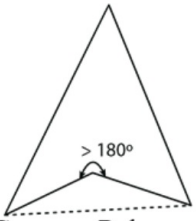
18. IF BG = (2x – 15), find x.

$25 = 2x - 15$
$+15 \quad +15$
$\frac{40}{2} = \frac{2x}{2}$
$x = 20$

Key Concepts:

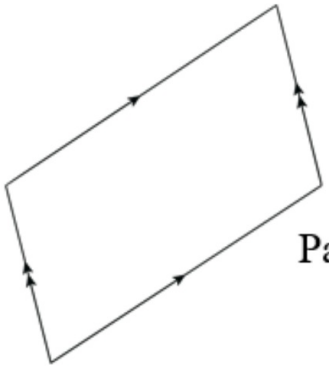
- A quadrilateral is a polygon with four sides.
- A Convex polygon is a polygon with no interior angle greater than 180° and all diagonals lie inside the polygon.
- A diagonal of a polygon is a line that connects Nonconsecutive vertices.
- Convex polygons are contrasted with concave polygons.
- A Concave polygon is a polygon with at least one interior angle greater than 180° and at least one diagonal that does not lie entirely inside the polygon.
- A parallelogram is a special type of quadrilateral with two pairs of opposite sides that are parallel.


Convex Polygon



Concave Polygon

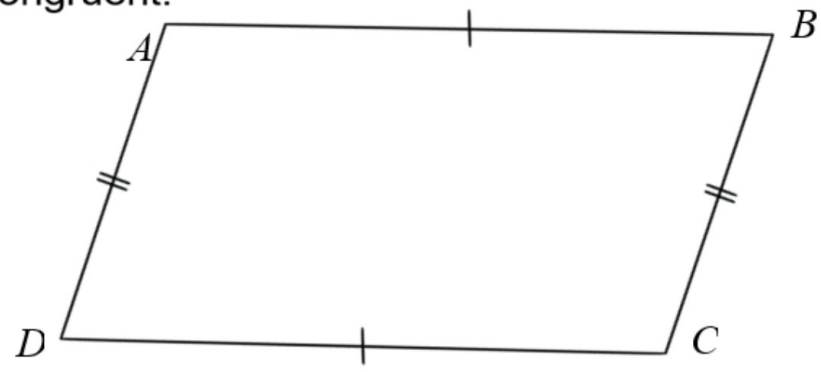
- By definition, if a quadrilateral has two pairs of opposite sides that are parallel, then the quadrilateral is a parallelogram.
- Parallelograms are denoted by the symbol $\square$.
- If a polygon is a parallelogram, there are five theorems associated with it.
- In a parallelogram, both pairs of opposite sides are congruent.



Parallelogram

Theorem

If a quadrilateral is a parallelogram, opposite sides are congruent.



$\overline{AB} \cong \overline{DC}$

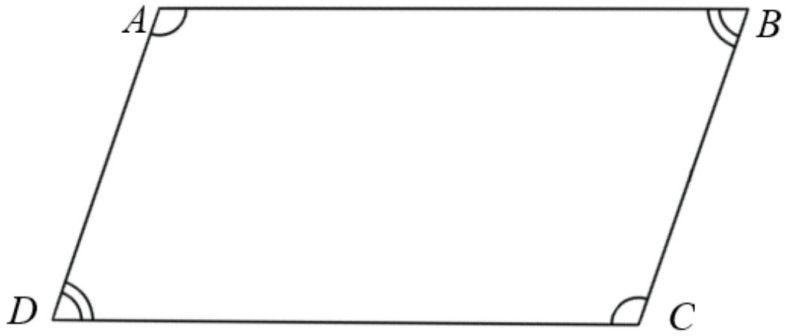
$\overline{AD} \cong \overline{BC}$

The converse is also true. If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

- Parallelograms also have two pairs of opposite angles that are congruent.

Theorem

If a quadrilateral is a parallelogram, opposite angles are congruent.



$\angle A \cong \angle C$

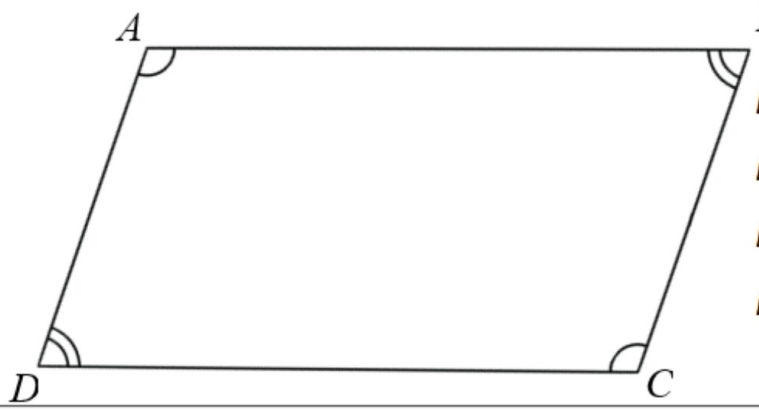
$\angle B \cong \angle D$

The converse is also true. If the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

- Consecutive angles are angles of a polygon that share a side.
- In a parallelogram, consecutive angles are supplementary; that is, they sum to 180°.

Theorem

If a quadrilateral is a parallelogram, then consecutive angles are supplementary.



$m\angle A + m\angle B = 180$

$m\angle B + m\angle C = 180$

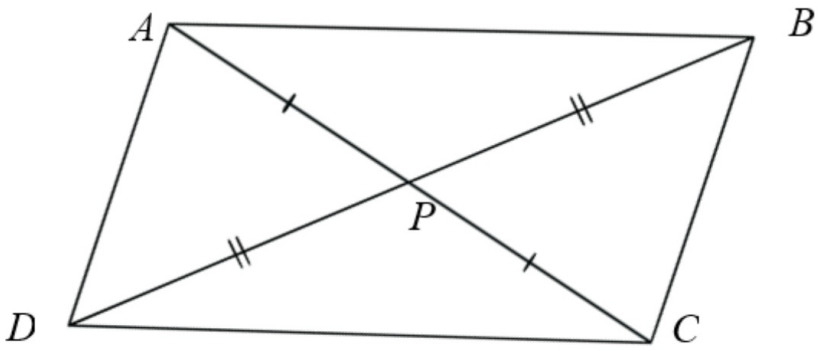
$m\angle C + m\angle D = 180$

$m\angle D + m\angle A = 180$

- The diagonals of a parallelogram have a relationship. They bisect each other.

Theorem

The diagonals of a parallelogram bisect each other.



$\overline{AP} \cong \overline{PC}$

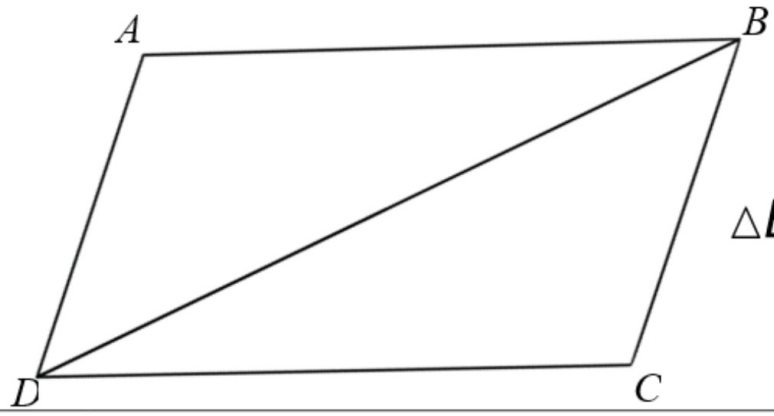
$BP \cong PD$

The converse is also true. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

- Notice that each diagonal divides the parallelogram into two triangles. Those two triangles are congruent.

Theorem

The diagonal of a parallelogram forms two congruent triangles.

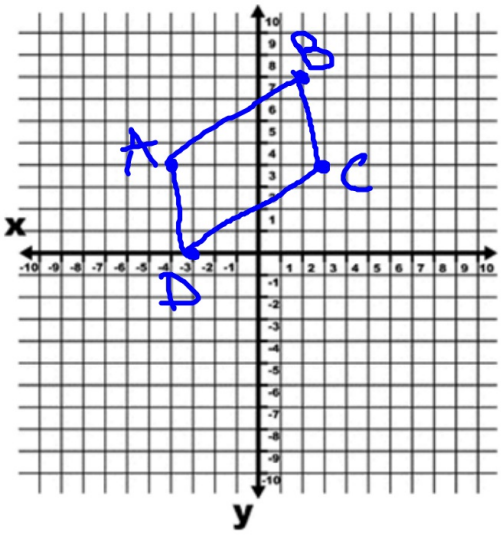


$\triangle BAD \cong \triangle DCB$

Example 1

Quadrilateral *ABCD* has the following vertices: *A* (–4, 4), *B* (2, 8), *C* (3, 4), and *D* (–3, 0). Determine whether the quadrilateral is a parallelogram. Verify your answer using slope and distance to prove or disprove that opposite sides are parallel and opposite sides are congruent.

1. **Graph the figure.**



$y = mx + b$

m ↑ Slope

b ↑ y-int.

2. **Determine whether opposite pairs of lines are parallel.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \frac{8-4}{2-(-4)} = \frac{4}{6} = \frac{2}{3}$ ✓

$m_{CD} = \frac{0-4}{-3-3} = \frac{-4}{-6} = \frac{2}{3}$

$m_{BC} = \frac{4-8}{3-2} = \frac{-4}{1} = -4$ ✓

$m_{AD} = \frac{0-4}{-3-(-4)} = \frac{-4}{1} = -4$

3. **Verify that the opposite sides are congruent.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(2-(-4))^2 + (8-4)^2}$
$= \sqrt{(6)^2 + (4)^2}$
$= \sqrt{36+16}$
$= \sqrt{52}$

$BC = \sqrt{(3-2)^2 + (4-8)^2}$
$= \sqrt{(1)^2 + (-4)^2}$

$DC = \sqrt{(3-(-3))^2 + (4-0)^2}$
$= \sqrt{(6)^2 + (4)^2}$
$= \sqrt{36+16}$
$= \sqrt{52}$

$AD = \sqrt{(-3-(-4))^2 + (0-4)^2}$
$= \sqrt{(1)^2 + (-4)^2}$