Math 3

-Warm-up #13-16
-Attendance
-Collect homework and late work
-3.4 Notes on Solving Polynomial Functions
-Homework on Synthetic Division and Solving Polynomial Functions

3.4 – Polynomial Functions of Higher Degree

Objectives

■ Find and use the real zeros of polynomial functions as sketching aids.

Real Zeros of Polynomial Functions

It can be shown that for a polynomial function *f* of degree *n*, the following statements are true.

1. The function *f* has, at most, *n* real zeros.

2. The graph of *f* has, at most, *n* – 1 turning points. (Turning points, also called relative minima or relative maxima, are points at which the graph changes from increasing to decreasing or vice versa.)

Finding the zeros of polynomial functions is one of the most important problems in algebra.

Real Zeros of Polynomial Functions

When f is a polynomial function and a is a real number, the following statements are equivalent.

1. $x = a$ is a *zero* of the function f.
2. $x = a$ is a *solution* of the polynomial equation $f(x) = 0$.
3. $(x - a)$ is a *factor* of the polynomial $f(x)$.
4. $(a, 0)$ is an *x-intercept* of the graph of f.

EXAMPLES

Find all real zeros of the polynomial function,

1) $f(x) = x^2 - 6x + 9$

9

-3 -3

-6

$(x-3)(x-3)$

$x-3=0$ $\quad$ $x-3=0$

$+3\ +3$ $\quad$ $+3\ +3$

$x=3$ $\quad$ $x=3$

X=3 multiplicity of 2

Mult. of 2

2) $g(x) = x^5 - 8x^3 + 16x$

$x(x^4 - 8x^2 + 16)$

$x(x^2 - 4)(x^2 - 4)$

16

-4×-4

-8

$x(x+2)(x-2)(x+2)(x-2)$

$x = 0$

$x + 2 = 0$

$-2 \quad -2$

$x = -2$ mult. of 2

$x - 2 = 0$

$+2 \quad +2$

$x = 2$ mult. of 2

3) $h(x) = x^3 + 3x^2 - 4x - 12$

4) $x^4 - 2x^3 - 4x^2 + 11x - 6 = 0$

5) $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

6) $h(x) = x^3 - 2x^2 + 2x - 1$

7) $p(x) = 2x^5 + 5x^4 - 8x^3 - 14x^2 + 6x + 9$

8) $x^4 - x^3 - 3x^2 + 17x - 30 = 0$

9) $3x^3 - 4x^2 - 2x + 3 = 0$