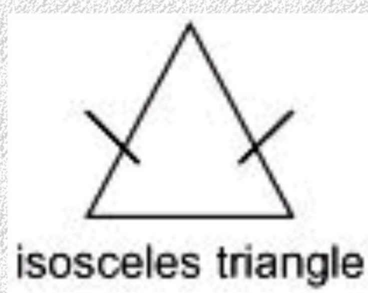
Isosceles triangles are common in the real world.

You find them in structures such as bridges and buildings.

# Isosceles Triangle

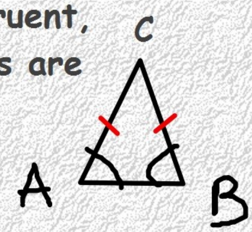
Two congruent sides



**Theorem 4-1** If two sides of a triangle are congruent, then the angles opposite those sides are also congruent.

**Isosceles Triangle Theorem**

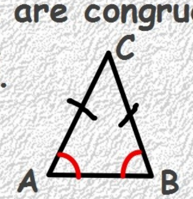
If $\overline{AC} \cong \overline{BC}$, then $\angle A \cong \angle B$.



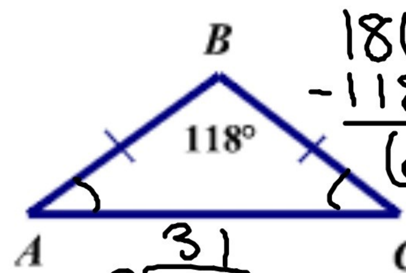

**Theorem 4-3** If two angles of a triangle are congruent, then the sides opposite the angles are congruent.

**Converse of Isosceles Triangle Theorem**

If $\angle A \cong \angle B$, then $\overline{AC} \cong \overline{BC}$.




d) $m\angle A =$ 31°

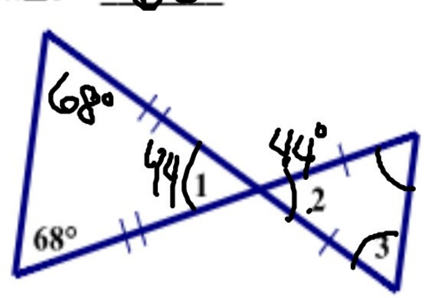



$$\begin{array}{r} 180 \\ -118 \\ \hline 62 \end{array}$$

$$2\overline{)62} = 31$$

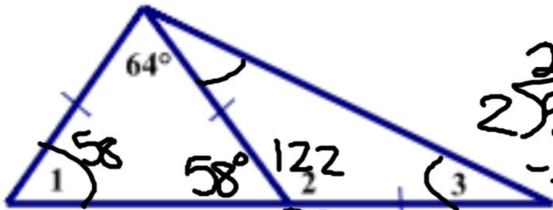
c) $m\angle 1 =$ 44 $m\angle 2 =$ 44

$m\angle 3 =$ 68°




$$\begin{array}{r} 180 \\ -136 \\ \hline 44 \end{array} \qquad \begin{array}{r} 68 \\ +68 \\ \hline 136 \end{array}$$

b) $m\angle 1 =$ 58° $m\angle 2 =$ 122°
$m\angle 3 =$ 29°



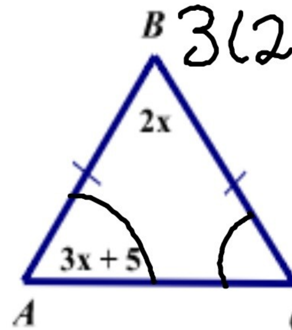

$$\begin{array}{r} 29 \\ 2\overline{)58} \\ -4 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 180 \\ -64 \\ \hline 116 \end{array} \quad \begin{array}{r} 58 \\ 2\overline{)116} \end{array} \quad \begin{array}{r} 180 \\ -58 \\ \hline 122 \end{array} \quad \begin{array}{r} 180 \\ -122 \\ \hline 58 \end{array}$$

$2(3x+5)+2x=180$
$6x+10+2x=180$
$8x+10=180$
$-10 \quad -10$
$8x=170$

c) $x =$ 21.25 $m\angle A =$ 68.75

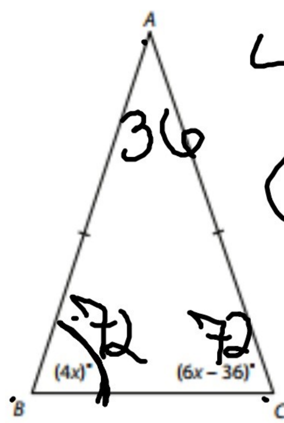
$3(21.25)+5$




**Example 1**

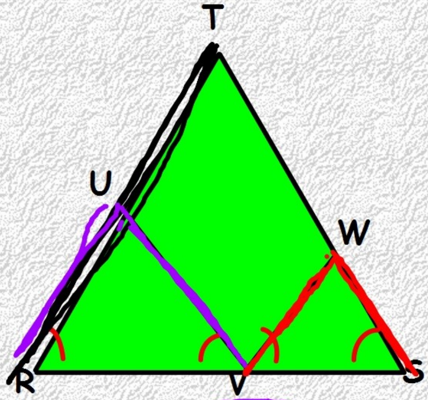
Find the measure of each angle of $\triangle ABC$ .

$4x=6x-36$
$-4x \quad -4x$
$0=2x-36$
$+36 \quad +36$
$\frac{36}{2}=\frac{2x}{2}$ $x=18$




$4(18)$
72°

$$\begin{array}{r} 72 \\ +72 \\ \hline 144 \end{array}$$




$\overline{RT} \cong$ $\overline{ST}$ $\overline{RU} \cong$ $\overline{UV}$ $\overline{VW} \cong$ $\overline{SW}$

$\overline{RT} \cong \overline{ST}$ because $\angle R \cong \angle S$

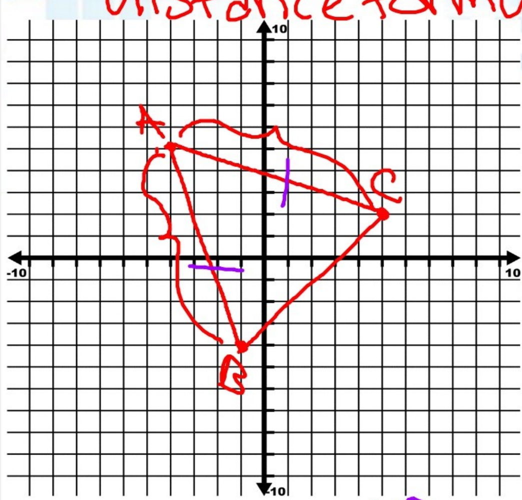
$\overline{RU} \cong \overline{VU}$ because $\angle R \cong \angle RVU$

$\overline{VW} \cong \overline{SW}$ because $\angle WVS \cong \angle S$

## Example 2

Determine whether $\triangle ABC$ with vertices $A$ (–4, 5), $B$ (–1, –4), and $C$ (5, 2) is an isosceles triangle. If it is isosceles, name a pair of congruent angles.

distance formula: $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$




$\overline{AB}$: $\sqrt{(-4+(+1))^2+(5+(+4))^2}$

$* = \sqrt{(-3)^2+(9)^2}$

$= \sqrt{9+81}$

$= \sqrt{90}$

$\overline{AC}$: $\sqrt{(5+(+4))^2+(2-(+5))^2}$

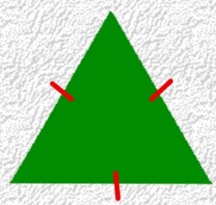
$* \sqrt{(9)^2+(-3)^2}$

14

$\angle B \cong \angle C$

# Quick Review

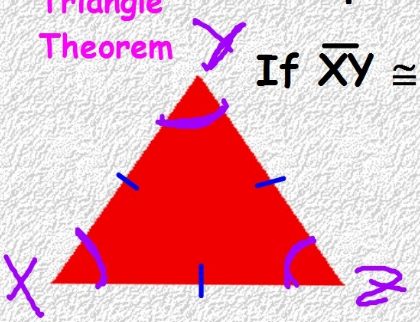
An equilateral triangle has three congruent sides. An equiangular triangle has three congruent angles. These words can be used interchangeably for triangles only.

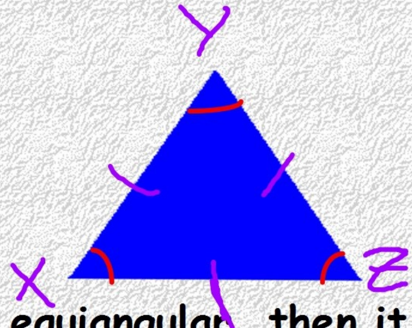


**Corollary** to Isosceles Triangle Theorem

If a triangle is equilateral, then it is equiangular.

If $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$, then $\angle X \cong \angle Y \cong \angle Z$.




**Corollary** to converse of Isosceles Triangle Theorem

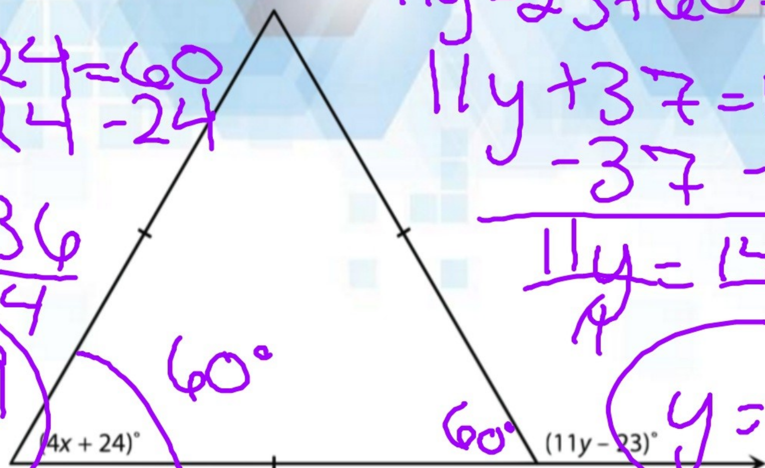
If a triangle is equiangular, then it is equilateral.

If $\angle X \cong \angle Y \cong \angle Z$, then $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$.

## Guided Practice

### Example 4

Find the values of *x* and *y*.

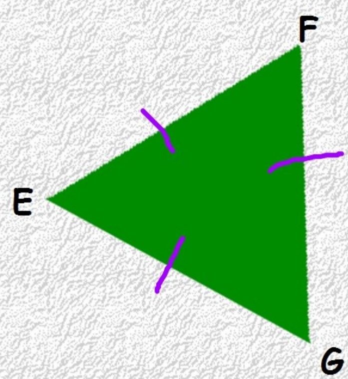



$$3\overline{)180}\quad 60$$

$$4x + 24 = 60$$
$$-24 \quad -24$$
$$\frac{4x}{4} = \frac{36}{4}$$
$$x = 9$$

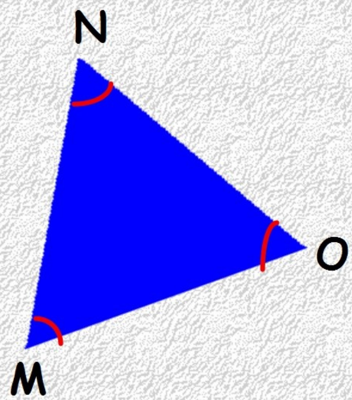
$$11y - 23 + 60 = 180°$$
$$11y + 37 = 180$$
$$-37 \quad -37$$
$$\frac{11y}{11} = \frac{143}{11}$$
$$y = 14$$

22




Choose two sides of $\Delta$EFG. What must be true about the angles opposite these sides? Why? All angles are equal because all sides are congruent

What is true about the angles of an equilateral triangle? 60°




Choose two angles of ΔMNO. What must be true about the sides opposite these angles? Why? the sides are equal because the angles are congruent.

What is true about the sides of an equiangular triangle? They are all congruent.