

WARM UP

Get out your homework

$$\frac{2x+1}{x^2+2x-8} + \frac{2}{x+4} = \frac{1}{x-2}$$

Handwritten work: $\frac{2x+1}{(x+4)(x-2)} + \frac{2(x-2)}{(x+4)(x-2)} = \frac{1}{x-2}$

$$2x+1 + 2x-4 = x+4$$
$$4x-3 = x+4$$
$$\begin{array}{r} +3 \quad +3 \\ \hline 4x = x+7 \\ -x \quad -x \\ \hline -x = -x+7 \end{array}$$
$$\frac{3x}{3} = \frac{7}{3}$$

$x = \frac{7}{3}$

OBJECTIVE: I can graph a rational function, identifying asymptotes, intercepts, and points.

We now know how to simplify and solve rational equations, but what do these functions look like when they are graphed. Let's explore this one briefly.

$$y = \frac{x-1}{x-2}$$

Those dashed lines on the graph are called the Asymptotes

The vertical one is $x = 2$, the horizontal one is $y = 1$.

What is significant about the vertical asymptote?

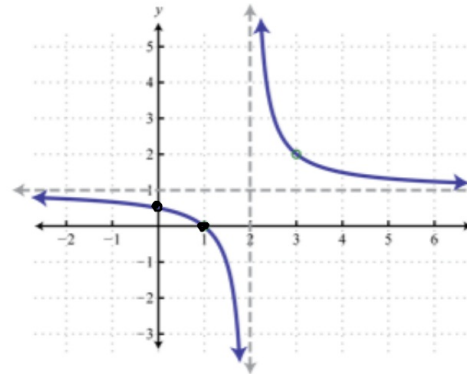
graph can never cross.

What is the x-intercept?

$$x = 1$$

What is the y-intercept?

$$y = .5$$



What are the two points we know exist on the graph?

What is the domain? $(1, 0), (0, .5)$
 $(-\infty, 2) \cup (2, \infty)$ What is the range? $(-\infty, 1) \cup (1, \infty)$

GUIDELINES FOR GRAPHING RATIONAL FUNCTIONS

- 1) Always start by factoring the numerator and denominator if needed.
- 2) Find your x and y intercepts.
- 3) Find any asymptotes.
- 4) Plot at least one point between the asymptotes and one beyond each vertical asymptote and x-intercept.

DOMAIN – Is all real numbers that x can be, except where the denominator equals zero. The graph will run against that line but never touch it.

RANGE – Is all the possible y-values of the function. Sometimes all real numbers, except the value of the horizontal asymptote (but not always).

X-INTERCEPT(S) – Find by setting the numerator equal to zero and solving. What value of x will make the numerator equal 0?

Y-INTERCEPT(S) – Find by plugging zero in for x and simplifying.

VERTICAL ASYMPTOTES – Set the denominator equal to zero and solve. What values for x will give you 0 in the denominator?

HORIZONTAL ASYMPTOTES

<p>If the numerator's degree is LESS THAN the denominator's, the horizontal asymptote is $y = 0$</p>	<p>If the numerator and denominator degree ARE EQUAL then $\frac{a}{b}$ is the horizontal asymptote where a and b are the lead coefficients of the numerator and denominator.</p>	<p>If the numerator's degree is GREATER THAN that of the denominator, there is no horizontal asymptote.</p>
<p>$f(x) = \frac{x}{x^2-9}$ OR $f(x) = \frac{3x^2+4}{x^3-8}$ HA = $y=0$ HA = $y=0$</p>	<p>$f(x) = \frac{x}{x-9}$ OR $f(x) = \frac{4x^2+1}{2x^2-3}$ HA = $y=1$ HA = $\frac{4}{2} y=2$</p>	<p>$f(x) = \frac{x^2}{x-9}$ OR $f(x) = \frac{x^5+1}{3x^2-3}$ HA = N/A HA = N/A</p>

PRACTICE:

$$f(x) = \frac{x}{x^2 - 2x - 3}$$

$$(x-3)(x+1)$$

HA: $y=0$

VA: $x=3$
 $x=-1$

$$\frac{-3}{2}$$

$$f(x) = \frac{x^3 + 5}{x^2 - 1}$$

$$(x+1)(x-1)$$

HA: N/A

VA: $x=-1$
 $x=1$

$$(x-3)(3x+4)$$

$$f(x) = \frac{2x^2 - 2x + 1}{3x^2 - 5x - 12}$$

HA: $y = \frac{2}{3}$

VA: $x=3$
 $x = \frac{-4}{3}$

$$-3 = \frac{-9}{3} \times \frac{4}{3}$$

EXAMPLE A: Graph the function and find the domain, range, vertical and horizontal asymptotes and x and y intercepts.

$$f(x) = \frac{3x}{x^2 - 4} = \frac{3x}{(x+2)(x-2)}$$

HA: $y=0$

VA: $x=2$
 $x=-2$

x-Intercept:

$$x=0$$

y-Intercept:

$$y=0$$

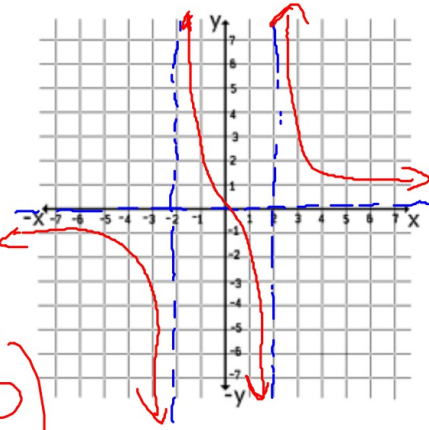
Domain:

$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

Range:

$$(-\infty, \infty)$$

x	f(x)



EXAMPLE B: Graph the function and find the domain, range, vertical and horizontal asymptotes and x and y intercepts.

$$f(x) = \frac{2x+1}{x-3}$$

HA: $y=2$ VA: $x=3$

x-Intercept:

$$x = -0.5$$

y-Intercept:

$$y = -\frac{1}{3}$$

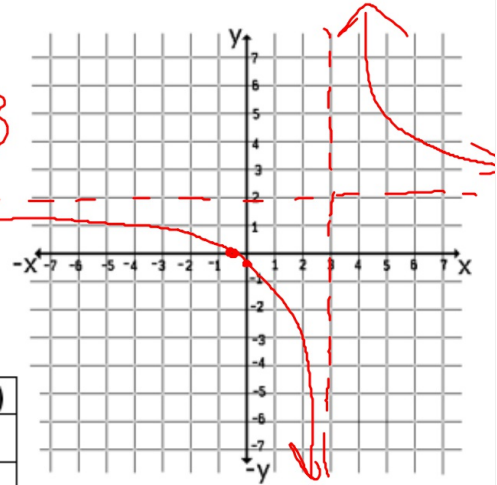
Domain:

$$(-\infty, 3) \cup (3, \infty)$$

Range:

$$(-\infty, 2) \cup (2, \infty)$$

x	f(x)



EXAMPLE C: Graph the function and find the domain, range, vertical and horizontal asymptotes and x and y intercepts.

$$f(x) = \frac{x^2 - 2x - 3}{2x^2 - x - 10} \quad f(x) =$$

HA:

VA:

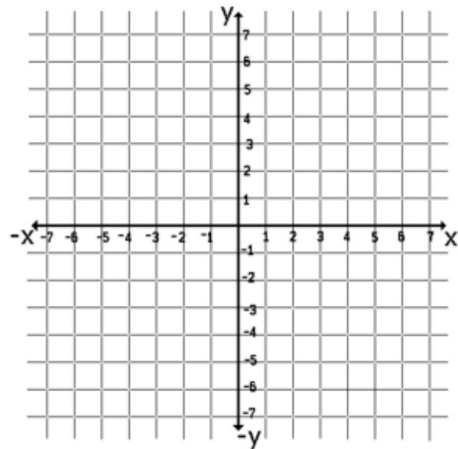
x-Intercept:

y-Intercept:

Domain:

Range:

x	f(x)



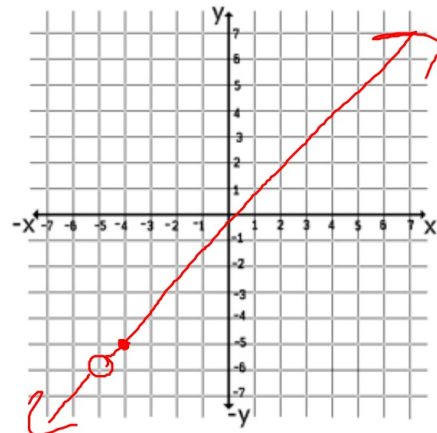
GUIDED NOTES – Lesson 5-4b

Discontinuity, Holes, and Applications

Sometimes when you go to graph rational equation, the problem can be simplified after factoring. This creates a point of discontinuity...

$$f(x) = \frac{x^2 + 4x - 5}{x + 5}$$

$5 \cancel{x} - 1$ (also known as a hole)
 $\frac{(x+5)(x-1)}{\cancel{x+5}}$
 $x \neq -5$ $x-1$



hole @ $x = -5$

$$f(x) = \frac{x^2 + 4x + 4}{x^2 - 4}$$

$\frac{2 \cancel{x} + 2}{\cancel{x} - 2}$ $\frac{(x+2)(x+2)}{(x+2)(x-2)}$
 $\frac{x+2}{x-2}$

Vertical Asymptote (x=):

$$x = 2$$

Horiz. Asymptote (y=):

$$x = -2 \text{ hole}$$

$$y = 1$$

x-Intercept:

(set numerator = to 0) $x = -2$

y-Intercept:

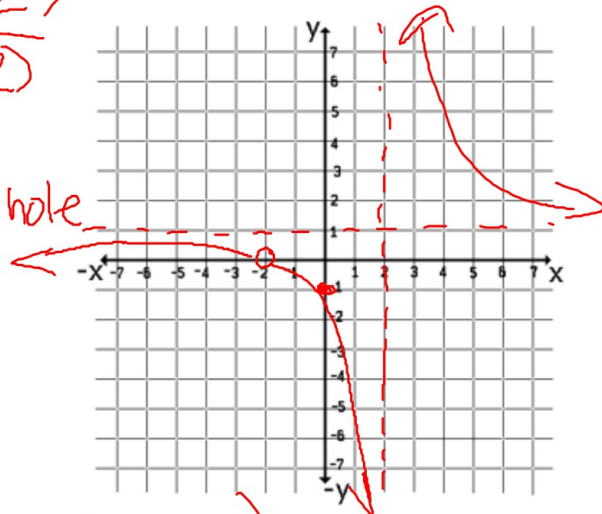
(plug in zero) $y = -1$

Domain:

$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

Range:

$$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$$



x	f(x)

Application: Ed plays football for his high school. So far this season, he has made 7 out of 11 field goals. He would like to improve his field goal percentage. If he can make x consecutive field goals, his field goal percentage can be determined by this function.

$$f(x) = \frac{x + 7}{x + 11}$$

Graph in your calculator & window: $X_{\min}=0$, $X_{\max} = 55$,
 $Y_{\min}=0$, $Y_{\max}=1$



What is the y-intercept?

What does it mean in the context?

What is the horizontal asymptote?

What does it mean in the context?

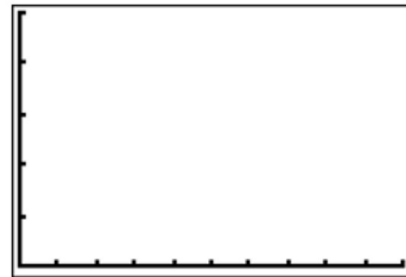
If he makes the next 5 kicks in a row, what will his field goal percentage be?

If he makes his next 50 kicks in a row, what will his field goal percentage be?

Application: The concentration of a drug is monitored in the bloodstream of a patient.

The drug's concentration can be modeled by $C(t) = \frac{5t}{t^2+1}$ where t is in hours and $C(t)$ is mg.

Graph in your calculator & window: $X_{\min}=0$, $X_{\max} = 10$,
 $Y_{\min}=0$, $Y_{\max}=5$



What is the horizontal asymptote?

Describe the drug's concentration in the patient's bloodstream over time.

What is the maximum concentration and when does it occur?

What is the concentration after 3 hours?

Will the drug ever leave the patient's bloodstream?