Math III

$a = b \quad a = -b$

...rk

...at is the solution to the equation $|3 - x| = 9$?

$3 - x = 9 \qquad 3 - x = -9$

$+x \quad +x \qquad +x \quad +x$

$3 = 9 + x \qquad 3 = -9 + x$

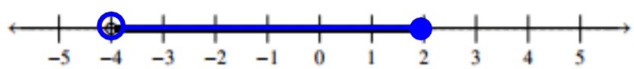
$-9 \quad -9 \qquad +9 \quad +9$

$x = -6 \qquad x = 12$

... 6 or −6   B. −6 or 12

... −12 or 6   D. no real solution

2. Which inequality is represented by the graph?



A. $-4 \le x < 2$ B. $-4 < x \ge 2$ C. $-4 < x < 2$ D. $-4 <$ ...

$< \ \le$

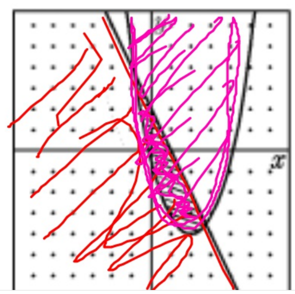
...hat is the equation of the system that would give the graph shown?

... $y \le -2x + 2$, $y \ge (x-2)^2 - 4$

B. $y \le -2x + 2$, $y \le (x-2)^2 - 4$

... $y \ge -2x + 2$, $y \ge (x-2)^2 - 4$

D. $y \ge 2x - 2$, $y \le (x-2)^2 - 4$



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1. $A(x) = \begin{cases} -x + 4, & x \le -2 \\ x - 2, & x > 2 \end{cases}$ ① ②

$-4 \le -2x$

$\cancel{-4 > 2}$

a. $A(-4)$: $-(-4) + 4$ = 8

b. $A(-2)$: $-(-2) + 4$ = 6

c. $A(0)$: ∅

d. $A(2)$: ∅

e. $A(5)$: $5 - 2 = 3$

2. $B(x) = \begin{cases} x - 3, & x < -6 \\ x^2, & -4 \le x < 3 \\ 7, & x > 3 \end{cases}$

a. $B(-7)$: $-7 - 3 = -10$

b. $B(-2)$: $-2^2 = 4$

c. $B(2)$: $2^2 = 4$

d. $B(3)$: ∅

e. $B(5)$: 7

3. $C(x) = \begin{cases} x + 2, & x \le -5 \\ |x - 3| + 4, & -5 < x < 3 \\ -x, & x > 3 \end{cases}$

a. $C(-6)$: $-6 + 2 = -4$

b. $C(-5)$: $-5 + 2$ = −3

c. $C(-2)$: $|-2-3| + 4$, $|-5| + 4$ = 9

d. $C(1)$: $|1-3| + 4$, $|-2| + 4$ = 6

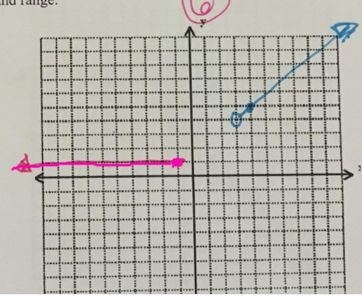
e. $C(4)$: −4

Questions 4 – 7, graph the piecewise function and state the domain and range.

4. $D(x) = \begin{cases} 1, & x \le -1 \\ x + 1, & x > 3 \end{cases}$

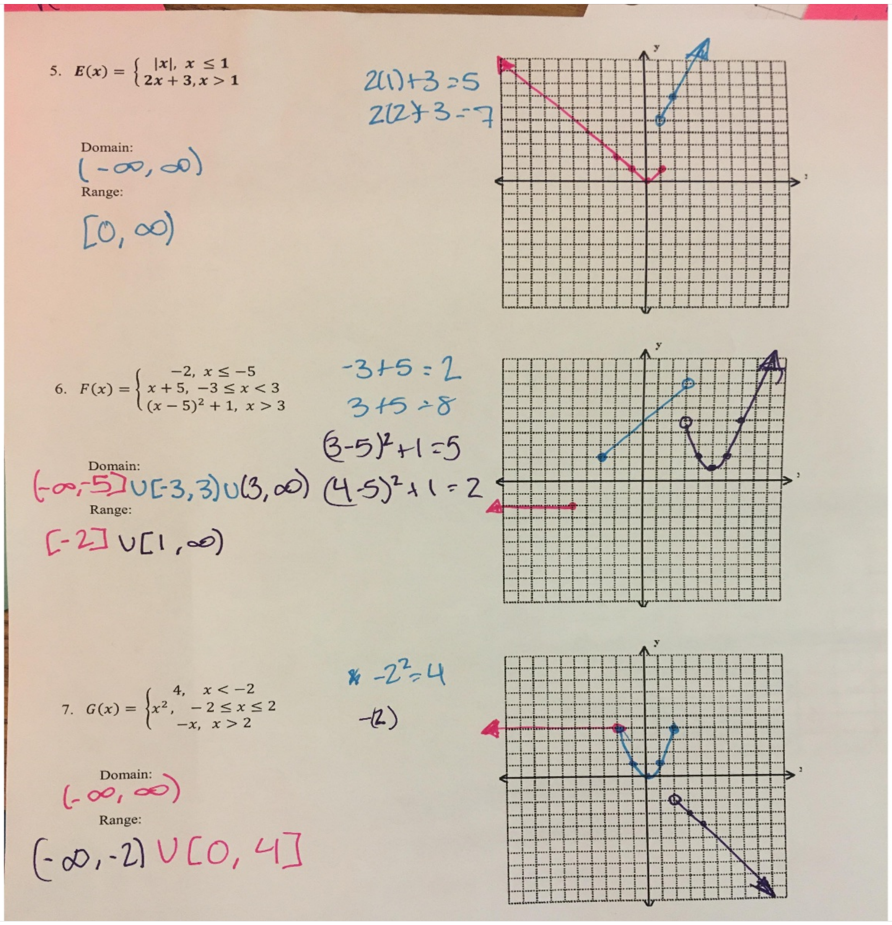
$3 + 1 = 4$

$4 + 1 = 5$



Domain: $(-\infty, -1] \cup (3, \infty)$

Range: $[1] \cup (4, \infty)$



5. $E(x) = \begin{cases} |x|, & x \leq 1 \\ 2x+3, & x > 1 \end{cases}$

$2(1)+3=5$
$2(2)+3=7$

Domain:
$(-\infty, \infty)$

Range:
$[0, \infty)$

6. $F(x) = \begin{cases} -2, & x \leq -5 \\ x+5, & -3 \leq x < 3 \\ (x-5)^2+1, & x > 3 \end{cases}$

$-3+5=2$
$3+5=8$
$(3-5)^2+1=5$
$(4-5)^2+1=2$

Domain:
$(-\infty,-5] \cup [-3,3) \cup (3,\infty)$

Range:
$[-2] \cup [1,\infty)$

7. $G(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \leq x \leq 2 \\ -x, & x > 2 \end{cases}$

$-2^2=4$
$-(2)$

Domain:
$(-\infty, \infty)$

Range:
$(-\infty,-2) \cup [0, 4]$

2.5 - Combinations of Functions: Composite Functions

Objectives

- Add, subtract, multiply, and divide functions.
- Find the composition of one function with another function.
- Use combinations and compositions of functions to model and solve real-life problems.

Arithmetic Combinations of Functions

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the *sum, difference, product,* and *quotient* of f and g are defined as follows.

1. Sum: $(f+g)(x) = f(x) + g(x)$
2. Difference: $(f-g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

EXAMPLE

$f(x) = 2x - 3$ and $g(x) = x^2 - 1$

Find

a) $(f+g)(x)$

$f(x)+g(x)$

$(2x-3)+(x^2-1)$

x^2+2x-4

b) $(f-g)(x)$

$f(x)-g(x)$

$(2x-3)-(x^2-1)$

$2x-3-x^2+1$

$\boxed{-x^2+2x-2}$

c) $(fg)(x)$

$f(x)\cdot g(x)$

$(2x-3)(x^2-1)$

$2x^3-2x-3x^2+3$

$\boxed{2x^3-3x^2-2x+3}$

d) $\left(\frac{f}{g}\right)(x)$

$\frac{f(x)}{g(x)}$

$\frac{2x-3}{x^2-1}$

$\boxed{\frac{2x-3}{(x+1)(x-1)} \quad x \neq -1, 1}$

Evaluating Arithmetic Combinations of Functions

EXAMPLE

$f(x) = x^2 - 1$ and $g(x) = x + 4$

Find

a) $(f+g)(2)$

$f(2)+g(2)$

$(2^2-1)+(2+4)$

$(4-1)+6$

$3+6 = 9$

b) $(f-g)(-3)$

$f(-3)-g(-3)$

$(-3^2-1)-(-3+4)$

$(9-1)-(1) = 7$

d) $(fg)(0)$

$f(0)\cdot g(0)$

$(0^2-1)(0+4) = (-1)(4)$

$= -4$

d) $\left(\frac{f}{g}\right)(7)$

$\frac{f(7)}{g(7)} = \frac{7^2-1}{7+4}$

$\frac{48}{11}$

e) $(fg)(0)+f(6)$

$-4+(6^2-1)$

$-4+(36-1)$

$-4+35 = 31$

Composition of Functions

Another way of combining two functions is to form the **composition** of one with the other.

For instance, if $f(x) = x^2$ and $\boxed{g(x) = x + 1}$, the composition of f with g is

$$f(g(x)) = \boxed{f(x + 1)}$$
$$= (x + 1)^2.$$

$f(g(x))$

This composition is denoted as $f \circ g$ and reads as "f composed with g."

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 2.36.)

EXAMPLE

Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, find the following.

$(x+2)(x+2)$
$x^2+2x+2x+4$
x^2+4x+4

$(4-x^2)(4-x^2)$
$16-4x^2-4x^2+x^4$
$16-8x^2+x^4$

a. $(f \circ g)(x)$

$f(g(x))$
$g(x) = 4-x^2$
$f(4-x^2) = (4-x^2)+2$
$\boxed{6-x^2}$

b. $(g \circ f)(x)$

$g(f(x))$
$f(x) = x+2$
$g(x+2) = 4-(x+2)^2$
$= 4-(x^2+4x+4)$
$\cancel{4}-x^2-4x-\cancel{4}$
$\boxed{-x^2-4x}$

c. $(g \circ g)(x)$

$g(g(x))$
$g(x) = 4-x^2$
$g(4-x^2) = 4-(4-x^2)^2$
$= 4-(16-8x^2+x^4)$
$= 4-16+8x^2-x^4$
$\boxed{= -12+8x^2-x^4}$

Evaluating Composition of Functions

$f(g(2))$

Find $(f \circ g)(2)$ for each set of composite functions.

1. $f(x) = x^2 + 1$ $\qquad g(x) = \sqrt{x}$

$g(2) = \sqrt{2}$
$f(\sqrt{2}) = (\sqrt{2})^2 + 1$
$= 2 + 1 = 3$

$\sqrt{2} \cdot \sqrt{2} = \sqrt{2 \cdot 2} = \sqrt{4} = 2$

2. $f(x) = |x|$ $\qquad g(x) = x + 6$

$f(g(2))$
$g(2) = 2+6$
$= 8$
$f(8) = |8| = 8$

3. $f(x) = \dfrac{1}{x}$ $\qquad g(x) = x + 3$

$f(g(2))$
$g(2) = 2+3$
$= 5$
$f(5) = \dfrac{1}{5}$

Application

$(4t+2)(4t+2)$

$16t^2+8t+8t+4$

$16t^2+16t+4$

The number N of bacteria in a refrigerated food is given by

$$N(T) = 20T^2 - 80T + 500, \quad 2 \le T \le 14$$

where T is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by

$20(16t^2+16t+4)$

$-80(4t+2)+500$

$$T(t) = 4t + 2, \quad 0 \le t \le 3$$

$320t^2 + \cancel{320t} + 80$

$\cancel{-320t} - 160 + 500$

where t is the time in hours.

a. Find the composition $(N \circ T)(t)$ and interpret its meaning in context.

$N(T(t)) = N(4t+2) = \boxed{20(4t+2)^2}\boxed{-80(4t+2)}\boxed{+500}$

$\boxed{320t^2+420}$

b. Find the time when the bacteria count reaches 2000.

The suggested retail price of a new hybrid car is p dollars. The dealership advertises a factory rebate of \$2000 and a 10% discount.

a) Write a function R in terms of p giving the cost of the hybrid car after receiving the rebate from the factory.

b) Write a function S in terms of p giving the cost of the hybrid car after receiving the dealership discount.

c) Form the composite functions $(R \circ S)(p)$ and $(S \circ R)(p)$ and interpret each.

d) Find $(R \circ S)(25{,}795)$ and $(S \circ R)(25{,}795)$. Which yields the lower cost for the hybrid car?