Math 3
Warm-Up

Use Synthetic Division to divide the polynomials

1.) $(x^3 - 13x^2 + 40x + 18) \div (x - 7)$

$$\begin{array}{r|rrrr} 7 & 1 & -13 & 40 & 18 \\ & \downarrow & 7 & -42 & -14 \\ \hline & 1 & -6 & -2 & 4 \end{array}$$

$x^2 - 6x - 2 + \frac{4}{x-7}$

2.) $(x^4 + 5x^3 - 6x + 3) \div (x + 3)$

$$\begin{array}{r|rrrrr} -3 & 1 & 5 & 0 & -6 & 3 \\ & \downarrow & -3 & -6 & 18 & -36 \\ \hline & 1 & 2 & -6 & 12 & -33 \end{array}$$

$x^3 + 2x^2 - 6x + 12 - \frac{33}{x+3}$

3.) $(8x^5 + 32x^4 + 5x + 20) \div (x + 4)$

$$\begin{array}{r|rrrrrr} -4 & 8 & 32 & 0 & 0 & 5 & 20 \\ & \downarrow & -32 & 0 & 0 & 0 & -20 \\ \hline & 8 & 0x^3 & 0x^2 & 0x & 5 & 0 \end{array}$$

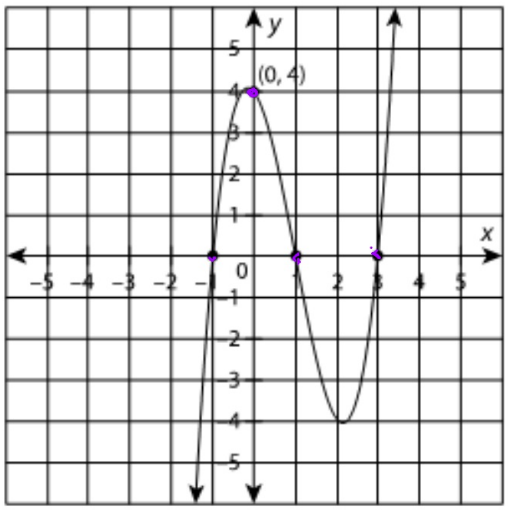
$8x^4 + 5$

Notes 3-4: Building Polynomials

Key Concepts:

- The solutions to polynomial functions are represented by the independent variable, when the dependent variable is equal to 0.
- Solutions can be represented as the x-intercepts of a graph, independent values in a table where the dependent value is equal to 0, in word problems, or as the algebraic solutions where factors are equal to 0.
- Polynomial functions can be built by determining the roots, building factors from the roots, multiplying the factors, and substituting a point to solve for the leading coefficient.

- To determine real solutions from a graph, locate the x-intercepts.
- If any roots are double roots, they are represented by an x-intercept that is a relative minimum or relative maximum on the graph. Those factors must be written and multiplied twice when distributing the factors.



- The solutions are –1, 1, and 3, and another point on the graph is (0, 4).

- To determine solutions from a table, find the independent values where the dependent value is equal to 0.

x	–3	–2	–1	0	1	2	3
y	–64	–20	0	4	0	–4	0

- The solutions are –1, 1, and 3, and another point that satisfies the equation is (0, 4). Note that it is not clear whether any of these are double solutions; in these types of situations, you'll need more information to determine what degree the polynomial has. Generally, you should build the polynomial with minimal degree.
- To determine the solutions from a word problem, determine the independent values when the dependent variable equals 0.

To determine the equation from the roots and another point algebraically:

Step 1: Write the equation in terms of the dependent variable, usually *y*, using the factors of the equation built from the roots. Leave a variable *a* in front of the first factor.
Step 2: Substitute another point on the function for the *x* and *y* variables to solve for *a*.
Step 3: Substitute *a* into the original equation and distribute the binomial factors to write the equation in standard form.

- To build a polynomial function using the regression feature of a graphing calculator, first determine if the equation is a quadratic, cubic, or quartic equation.

- Using the Fundamental Theorem of Algebra, an equation with 2 solutions is quadratic, 3 solutions is cubic, and 4 solutions is quartic. The solution can be found where the polynomial is equal to 0.
- If at least one point other than the solutions is given, the calculator can perform an accurate regression.
- To find the real zeros of a polynomial function on a calculator, graph the function and determine the values of the *x*-intercepts.

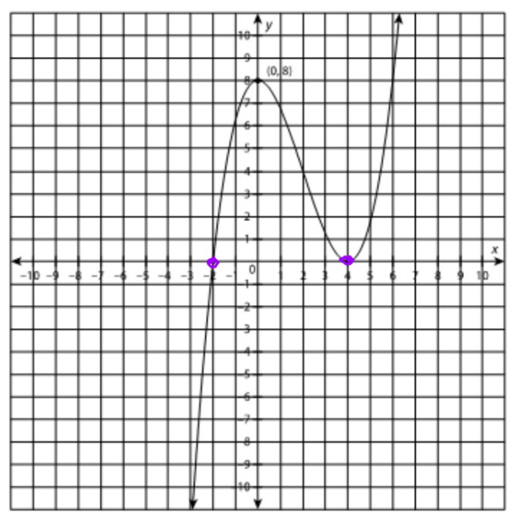
On a TI-83/84:

- **Step 1:** Determine the real roots of the function and the other point you can determine.
- **Step 2:** Press [STAT][EDIT] and select 1: Edit. Enter *x*-values in L_1 of the table and *y*-values in L_2.
- **Step 3:** Press [STAT][CALC]. If the equation is quadratic, select 5: QuadReg. If the equation is cubic, select 6: CubicReg. If the equation is quartic, select 7: QuartReg. Press "Calculate", and the coefficients of the terms in the polynomial will be calculated.
- **Step 4:** Substitute the coefficients before the variables in the standard form of the polynomial .

Guided Practice:

Example 1

Determine the equation that represents the graph at right using algebraic steps.



1. **Determine the roots of the equation from the graph.**

(-2, 0)

(4, 0) mult. of 2

2. **Name another point on the graph.** (0, 8)

3. Set up and solve the equation using the factors determined from the roots, the variable *a*, and the (*x*, *y*) coordinates from the other point to find *a*.

$x=-2 \rightarrow (x+2)(x-4)(x-4)$
$+2 \quad +2$

$x=4$
$-4 \quad -4$

$y=a(x+2)(x-4)(x-4)$

$8=a(0+2)(0-4)(0-4)$

$8=a(2)(-4)(-4)$

$\frac{8}{32}=\frac{a(32)}{32}$

$a=\frac{1}{4}$

4. Substitute the value of *a* back into the equation for the graph, and distribute to write the polynomial in standard form.

$y=\frac{1}{4}(x+2)(x-4)(x-4)$

$y=\frac{1}{4}(x^2-4x+2x-8)(x-4)$

$y=\frac{1}{4}(x^2-2x-8)(x-4)$

$y=\frac{1}{4}(x^3-4x^2-2x^2\cancel{+8x}\cancel{-8x}+32)$

$-6\cdot\frac{1}{4}=\frac{-6}{4}$

$32\cdot\frac{1}{4}=\frac{32}{4}$

$y=\frac{1}{4}(x^3-6x^2+32)$

$y=\frac{1}{4}x^3-\frac{3}{2}x^2+8$

degree: 3

$x \rightarrow -\infty, f(x) \rightarrow +\infty$

$x \rightarrow +\infty, f(x) \rightarrow -\infty$

2) degree: 4

$x \rightarrow -\infty, f(x) \rightarrow +\infty$

$x \rightarrow +\infty, f(x) \rightarrow +\infty$