Math 3
Warm-up

$10^x = 3 \rightarrow \log_{10} 3 =$

Find the inverse of each function.

1) $y = \log(-2x)$

$x = \log(-2y)$

$10^x = -2y$

$y = \frac{10^x}{-2}$

2) $y = \log_{\frac{1}{4}} x^5$

$x = \log_{\frac{1}{4}} y^5$

$\left(\frac{1}{4}\right)^x = y^5$

$\left(\frac{1}{4}\right)^{\frac{x}{5}} = y$

3) $y = \log_{\frac{1}{5}} x - 4$

$x = \log_{\frac{1}{5}} y - 4$

$+4 \quad +4$

$x + 4 = \log_{\frac{1}{5}} y$

$\left(\frac{1}{5}\right)^{x+4} = y$

4) $y = \log_3 (4^x - 4)$

$x = \log_3 (4^y - 4)$

$3^x = 4^y - 4$

$+4 \quad +4$

$3^x + 4 = 4^y$

$\log_4 3^x + 4 = y$

Remember that logarithmic equations can be rewritten as exponential equations. We will do this conversion in order to make solving logarithmic equations easier.

$\log_b x = y \rightarrow b^y = x$

Examples: Solve the following equations.

$\log_5 x = 2$

$5^2 = x$

$x = 25$

$\log_8 x = \frac{4}{3}$

$8^{4/3} = x$

$16 = x$

$\log_x 81 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{81}$

$x = 3\sqrt[3]{3}$

$9 \cdot 9$

$3 \cdot 3 \sqrt[3]{3}$

$\log_x 10 = 2$

$\sqrt{x^2} = \sqrt{10}$

$x = \sqrt{10}$

$log_2(x-4)=3$

$2^3 = x-4$

$8 = x-4$

$+4 \quad +4$

$x = 12$

$log_8(4x-3)=\frac{2}{3}$

$8^{2/3} = 4x-3$

$4 = 4x-3$

$+3 \quad +3$

$7 = 4x$

$\frac{7}{4} = x$

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If $\log_b M = \log_b N$, then... $M = N$

Examples: Solve the following equations.

$\log_6 x = \log_6 9$

$x = 9$

$\log_2 3x = \log_2 5(x-2)$

$3x = 5(x-2)$

$3x = 5x - 10$

$-5x \quad -5x$

$\frac{-2x}{-2} = \frac{-10}{-2}$

$x = 5$

$\log_4(5x-4) = \log_4(3x)$

$5x - 4 = 3x$

$+4 \quad +4$

$5x = 3x + 4$

$-3x \quad -3x$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

$\log_2(x^2-4) = \log_2 3x$

$x^2 - 4 = 3x$

$-3x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4 \quad x = -1$

$\log_3(x^2 - 3x - 19) = \log_3(x+4)$

$x^2 - 3x - 19 = x + 4$

$-x \quad -4 \quad -x - 4$

$x^2 - 4x - 23 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{4 \pm \sqrt{(-4)^2 - 4(1)(-23)}}{2(1)}$

$\frac{4 \pm \sqrt{16 + 92}}{2} = \frac{4 \pm \sqrt{108}}{2}$

108

$12 \cdot 9$

$4 \cdot 3 \quad 3 \cdot 3$

$2 \cdot 2$

$\frac{4 \pm 6\sqrt{3}}{2} = 2 \pm 3\sqrt{3}$

Using Logarithms to Isolate a Variable Exponent

1. Use inverse operations to isolate the base and exponent on one side of the equation.
2. Convert the exponential equation to a logarithm using the necessary conversion.
3. If necessary, solve the equation to isolate the variable.

Base 10	Base *e*	Base *b*
$10^x = a \rightarrow \log a = x$	$e^x = a \rightarrow \ln a = x$	$b^x = a \rightarrow \log_b a = x$

- When the base is 10 or *e*, you can use your calculator to find a decimal approximation by using the "log" or "ln" buttons. When the base is different, the change of base formula can be applied to write the base in the form of one of the common logarithms with base 10 or *e*. Then the logarithm can be evaluated for a decimal approximation in the calculator when necessary. The formula for this is

$$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}.$$

Example: Solve $6^{3x-8} = 6^{5x-17}$ for x.

$3x-8=5x-17$

$+8 \quad +8$

$3x=5x-9$

$-5x \quad -5x$

$-2x=-9$

$\frac{-2x}{-2}=\frac{-9}{-2}$

$x=\frac{9}{2}$

Solve $6(10)^{3x}-4=200$ for x.

$+4 \quad +4$

$\frac{6(10^{3x})}{6}=\frac{204}{6}$

$10^{3x}=34$

$\frac{\log_{10} 34}{3}=\frac{3x}{3}$

$x=.51$