Math 3

Warm-up

Find the inverse function

5) $y = 2^x$

$x = 2^y \rightarrow \log_2 x = y$

6) $y = 6^x$

$x = 6^y$

$\log_6 x = y$

7) $y = 10^{\frac{x}{3}}$

$x = 10^{\frac{y}{3}}$ $3(\log_{10} x) = \left(\frac{y}{\cancel{3}}\right)\cancel{3}$

$3\log_{10} x = y$

8) $y = 4^x + 8$

$x = 4^y + 8$

$-8 \qquad -8$

$x - 8 = 4^y$

$\log_4 (x-8) = y$

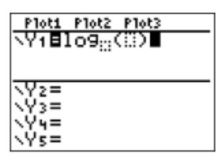
GUIDED NOTES – Lesson 5.2/5.3

Graphing Logarithmic Functions/Compound Interest & Growth/Decay

OBJECTIVE: I can identify the types of exponential functions, as well as evaluate and graph them.

LOGARITHMIC FUNCTION

$2^y = x \rightarrow log_2 x = y$



For the parent function, $y = log_b x$, the graph contains the ordered pairs (1, 0) and (b, 1). It has an asymptote at x = 0.

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(1, 0)$

y-intercept: N/A

TRANSFORMATIONS: $f(x) = (\underline{a})log_b(x \underline{-} \underline{h}) + \underline{k}$

h tells us about horizontal movement. *opposite sign

If **h is positive**... move right

If **h is negative**... move left

a tells us about stretching, reflecting, and compressing.

If **a is negative…** reflection

If **a > 1…** stretch

If **0 < a < 1…** Compress

k tells us about vertical movement.

If **k is positive…** Shift up

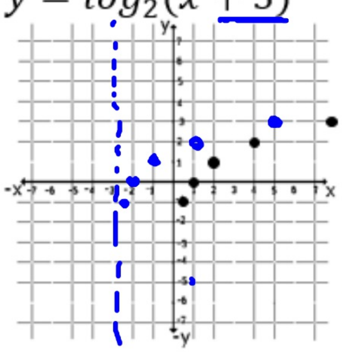
If **k is negative…** Shift down

So to graph logarithmic functions with transformations…

1. Plot the parent function ordered pairs and asymptote. (1, 0) and (b, 1)
2. Move each ordered pair and the asymptote h units and k units

$h = -3$

$y = log_2(x + 3)$



Domain: $(-3, \infty)$

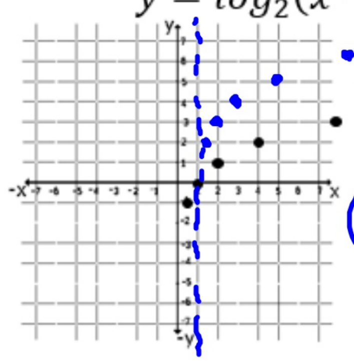
Range: $(-\infty, \infty)$

x-intercept: $(-2, 0)$

y-intercept: $(0, 1.5)$

$h = 1$
$k = 3$

$y = log_2(x - 1) + 3$



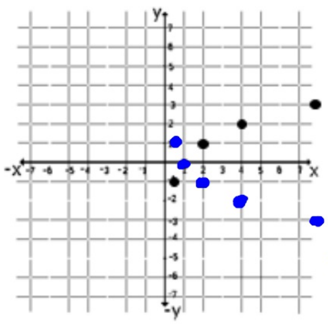
Domain: $(1, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(1.1, 0)$

y-intercept: N/A

$y = -log_2 x$ $a = -1$



Domain: $(0, \infty)$

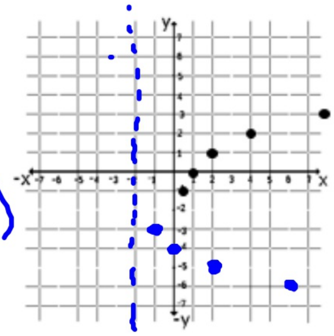
Range: $(-\infty, \infty)$

x-intercept: $(1, 0)$

y-intercept: N/A

$n = -2$

$y = -log_2(x+2) - 3$



Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(-1.875, 0)$

y-intercept: $(0, -4)$

OBJECTIVE: I can apply compound interest formulas and calculate growth and decay in real-world problems.

COMPOUND INTEREST: The the balance __A__ in an account with principal __P__ and annual interest rate __r__ (in decimal form) is given by the following formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$.5 \rightarrow 50\%$

For __n__ compounding per year after __t__ years.

Example A: Find the account balance after 20 years if $100 is placed in an account that pays 1.2% interest compounded twice a month.

P = $100

$r = 1.2\% \rightarrow .012$

$n = 24 \; (2 \times 12)$

$t = 20$

$$A = 100\left(1 + \frac{.012}{24}\right)^{24(20)}$$

$127.12

Example B: If \$350,000 is invested at a rate of 5% per year, find the amount of the investment at the end of 10 years for the following compounding methods:

$P = 350{,}000$
$r = 5\% \Rightarrow .05$
$t = 10$

a) Quarterly

$n = 4$

$A = 350{,}000\left(1+\frac{.05}{4}\right)^{4(10)}$

\$575,266.81

b) Monthly

$n = 12$

$A = 350{,}000\left(1+\frac{.05}{12}\right)^{12(10)}$

\$576,453.32

CONTINUOUSLY COMPOUNDED INTEREST: This is a hypothetical form of compounding, where the interest is computed and added to the balance of an account every instant (i.e. continuously).

$$A = Pe^{rt}$$

$$growth = e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Example C: Joan was born and her parents deposited \$2000 into a college savings account paying 4% interest compounded continuously. What would be the balance after 15 years?

$P = 2000$
$r = 4\% \Rightarrow .04$
$t = 15$

$A = 2000e^{.04(15)}$

\$3644.24

EXPONENTIAL GROWTH/DECAY: We learned in lesson 6-1 that exponential functions can take on two forms, either growth, when b is greater than 1 or decay, where b is between 0 and 1.

$$y = ab^x$$

y = final amount **a = initial amount** **b = growth/decay** **x = time/trials**

Example D: You are reading a novel where an entire 40 player baseball team become zombies. It is predicted that the number of zombies will triple each day. How many zombies will there be after a week (7 days)?

$a = 40$
$b = 3$
$x = 7$

$y = 40(3^7)$

87,480 zombies

Seems easy right, but you need to be careful with the variable b, because that example was a nice clean whole number.

Example E: You bought a used car for $18,000. The value of the car will be less each year because of depreciation. The car depreciates (loses value) at the rate of 12% per year. Write an exponential decay model to represent the situation then use that model to estimate the value of the car in 8 years.

$A = 18{,}000(1-.12)^8$

DECAY: $A = P(1-r)^t$

$P = 18{,}000$

$r = .12$

$t = 8$

$6473.42

Example F: A train is going downhill at 140 mph. Suddenly the brake system fails and the train begins picking up speed, going 11% faster every minute. How fast will the train be going in 5 minutes?

$P = 140$

$r = .11$

$t = 5$

$A = 140(1+.11)^5$

236 mph

GROWTH: $A = P(1+r)^t$